OPTIMAL MAINTENANCE OF A MULTI-UNIT SYSTEM

UNDER DEPENDENCIES

A Thesis
Presented to
The Academic Faculty

by

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In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy in the
School of Aerospace Engineering

Georgia Institute of Technology
December 2008

UMI Number: 3346076

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To my loving family…

ACKNOWLEDGEMENTS

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LIST OF ACRONYMS

$A(t)$	Long run average availability with respect to decision variable t
ALCCA	Aircraft Life Cycle Cost Analysis
ALT	Accelerated Life Test
CCDI	Central Composite Design Inscribed
CI	Confidence Interval
CM	Corrective Maintenance
CPU	Central Processing Unit
CTMC	Continuous Time Markov Chain
DFR	Decreasing Failure Rate
DOE	Design Of Experiments
FADEC	Full Authority Digital Engine Control
GA	Genetic Algorithm
GTRC	Georgia Tech Research Corporation
IDB	Increasing Decreasing Bathtub
IFR	Increasing Failure Rate
IFRA	Increasing Failure Rate in Average
$L(t)$	Long run average cost rate with respect to decision variable t
LOTC	Loss Of Thrust Control
LT	Long Term
MC	Monte Carlo
MDT	Mean Down Time

MEL	Minimum Equipment List
MLE	Maximum Likelihood Estimation
MTBF	Mean Time Between Failure
NHPP	Non-Homogeneous Poisson Process
OM	Opportunistic Maintenance
OMP	Optimal Maintenance Problem
PIR	Periodic Inspection / Repair
PM	Preventive Maintenance
RBD	Reliability Block Diagram
RCM	Reliability Centered Maintenance
ROP	Redundancy Optimization Problem
RSE	Response Surface Equation
RSM	Response Surface Methodology
SA	Simulated Annealing
SPN@	Stochastic Petri-Net with Aging token
SQP	Sequential Quadratic Programming
SSE	Sum Square of Error
ST	Short Term
TLD	Time Limited Dispatch
TSF	Time Since Failure
VRT	Variance Reduction Technique

SUMMARY

The availability, or reliability, of an engineering component greatly influences the operational cost and safety characteristics of a modern system over its life-cycle. Until recently, the reliance on past empirical data has been the industry-standard practice to develop maintenance policies that provide the minimum level of system reliability. Because such empirically-derived policies are vulnerable to unforeseen or fast-changing external factors, recent advancements in the study of topic on maintenance, which is known as optimal maintenance problem, has gained considerable interest as a legitimate area of research. An extensive body of applicable work is available, ranging from those concerned with identifying maintenance policies aimed at providing required system availability at minimum possible cost, to topics on imperfect maintenance of multi-unit system under dependencies.

Nonetheless, these existing mathematical approaches to solve for optimal maintenance policies must be treated with caution when considered for broader applications, as they are accompanied by specialized treatments to ease the mathematical derivation of unknown functions in both objective function and constraint for a given optimal maintenance problem. These unknown functions are defined as reliability measures in this thesis, and theses measures (e.g., expected number of failures, system renewal cycle, expected system up time, etc.) do not often lend themselves to possess closed-form formulas. It is thus quite common to impose simplifying assumptions on input probability distributions of components' lifetime or repair policies. Simplifying the complex structure of a multi-unit system to a k-out-of-n system by neglecting any sources

of dependencies is another commonly practiced technique intended to increase the mathematical tractability of a particular model.

This dissertation presents a proposal for an alternative methodology to solve optimal maintenance problems by aiming to achieve the same end-goals as Reliability Centered Maintenance (RCM). RCM was first introduced to the aircraft industry in an attempt to bridge the gap between the empirically-driven and theory-driven approaches to establishing optimal maintenance policies. Under RCM, qualitative processes that enable the prioritizing of functions based on the criticality and influence would be combined with mathematical modeling to obtain the optimal maintenance policies.

Where this thesis work deviates from RCM is its proposal to directly apply quantitative processes to model the reliability measures in optimal maintenance problem. First, Monte Carlo (MC) simulation, in conjunction with a pre-determined Design of Experiments (DOE) table, can be used as a numerical means of obtaining the corresponding discrete simulated outcomes of the reliability measures based on the combination of decision variables (e.g., periodic preventive maintenance interval, trigger age for opportunistic maintenance, etc.). These discrete simulation results can then be regressed as Response Surface Equations (RSEs) with respect to the decision variables. Such an approach to represent the reliability measures with continuous surrogate functions (i.e., the RSEs) not only enables the application of the numerical optimization technique to solve for optimal maintenance policies, but also obviates the need to make mathematical assumptions or impose over-simplifications on the structure of a multi-unit system for the sake of mathematical tractability.

The applicability of the proposed methodology to a real-world optimal

maintenance problem is showcased through its application to a Time Limited Dispatch (TLD) of Full Authority Digital Engine Control (FADEC) system. In broader terms, this proof-of-concept exercise can be described as a constrained optimization problem, whose objective is to identify the optimal system inspection interval that guarantees a certain level of availability for a multi-unit system. A variety of reputable numerical techniques were used to model the problem as accurately as possible, including algorithms for the MC simulation, imperfect maintenance model from quasi renewal processes, repair time simulation, and state transition rules. Variance Reduction Techniques (VRTs) were also used in an effort to enhance MC simulation efficiency. After accurate MC simulation results are obtained, the RSEs are generated based on the goodness-of-fit measure to yield as parsimonious model as possible to construct the optimization problem.

Under the assumption of constant failure rate for lifetime distributions, the inspection interval from the proposed methodology was found to be consistent with the one from the common approach used in industry that leverages Continuous Time Markov Chain (CTMC). While the latter does not consider maintenance cost settings, the proposed methodology enables an operator to consider different types of maintenance cost settings, e.g., inspection cost, system corrective maintenance cost, etc., to result in more flexible maintenance policies. When the proposed methodology was applied to the same TLD of FADEC example, but under the more generalized assumption of strictly Increasing Failure Rate (IFR) for lifetime distribution, it was shown to successfully capture component wear-out, as well as the economic dependencies among the system components.

CHAPTER 1   INTRODUCTION

The motivation for the research and the objective goal are discussed in this chapter. The steps to achieve the objective of the study are also addressed.

1.1   *Motivation*

There has been more and more emphasis on the reliability or availability in aircraft and rotorcraft design. The reliability is to capture the probability of the item working at given time, and the measure is used in nonrepairable items. On the other hand, the availability is used for repairable items to denote the proportion of uptime over the total operational time. Availability measure is discussed more often in this thesis, since most applications in aerospace engineering are repairable systems. There are accidents from the low availability of components which resulted in the catastrophic failures on the system. For example, US Air Boeing 737 crashed in Pennsylvania due to uncommanded deflection of the rudder to lose the control in 1994 [74], and TWA Flight 800 burned in Atlantic Ocean by the explosion of the center fuel tank in 1996 [73]. Besides the tragic accidents, many minor problems force the vehicle to make an emergency landing to avoid any further consequences as observed from Jet Blue accident by 90 degrees cocked nose wheel in 2005 [72]. Safe landing with minor problems would reduce the risk of a fatal accident, but it still costs a lot of operational expense to the operators. It has been discussed recently that commercial airliners try to outsource the maintenance to reduce the higher maintenance cost [45]. Theoretical relationship between reliability and operational expense for manufacturer and operator, such as airliner, is well documented

in [92]. The relationship implies the reason why reliability and availability assessment are closely related with the cost analysis under the process development as shown in Figure 1. The iterations between product and process development achieve a better design to meet the customer requirements.
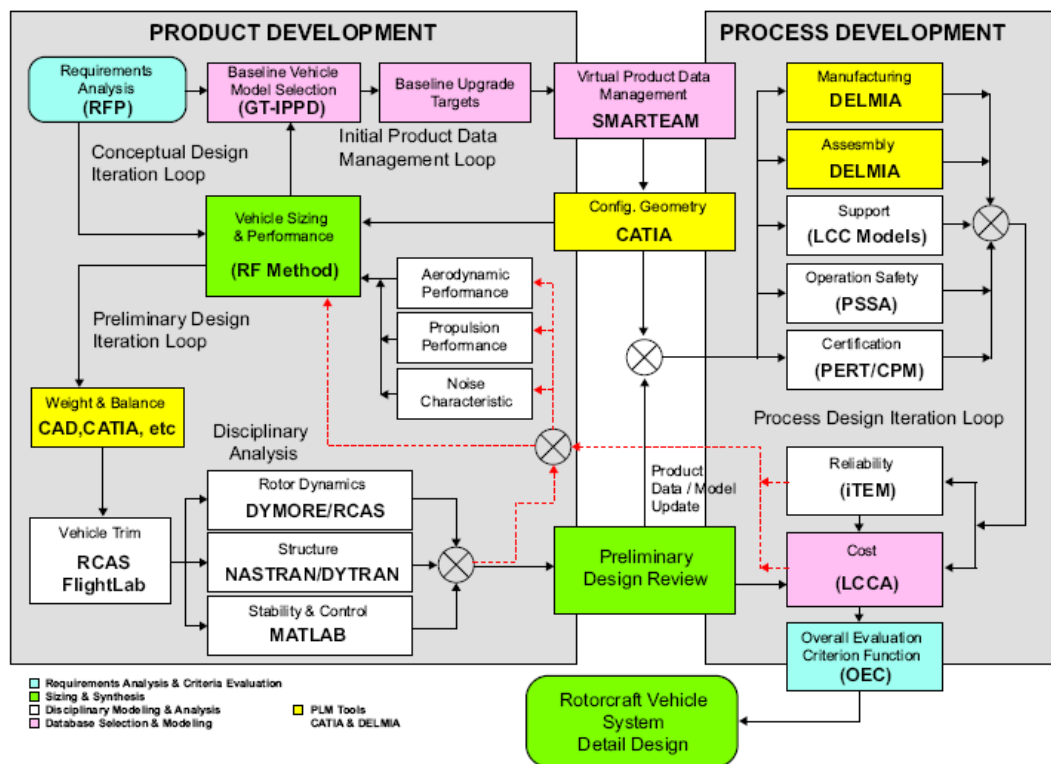
Figure 1: GT Vehicle Design Framework

It is observed that the framework does not provide the improvement in availability of the system. Generally, the framework helps to understand availability, cost or other metrics from process engineering as the design evolves. Of course, there are some papers which set metrics from process engineering as constraints for the vehicle design optimization [55]. Two approaches have been developed to improve the availability of the system directly. One is the redundancy allocation or optimization, and other is

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optimal maintenance problem.

Redundancy allocation or optimization searches for optimal combination of desired number and arrangement of redundancies. Generally, the objective function for the redundancy optimization problem (ROP) is the cost (weight) function of the system. In summary, ROP tries to find the combination of redundancies which gives the minimum objective function while achieving the constraints such as desired reliability or availability [95]. A lot of research has been done on ROP from multi state systems to stochastic variable cases where costs (weights) are random [84]. Overall survey of ROP is well discussed in [57].

Optimal maintenance problem approach is different from ROP because it starts with a given system. Operators solve ROP to construct the system with optimal combination of the redundancies to meet the requirement. On the other hand, solving optimal maintenance problem results in the best maintenance policy for a given system. It is natural to regard reliability of a component as a function of time. The mechanical or electrical components age with the passage of time, and they tend to have higher failure probabilities. Solving optimal maintenance problem sets the maintenance schedule so that proper maintenance on the components will result in desirable system availability as time passes. Similar to ROP, general setting for objective function is cost function. Further description on the optimal maintenance problem is discussed in Chapter 2.

Both approaches can improve reliability or availability of the system, but there is a limitation in implementing ROP to aerospace applications. Even though reliability of aircraft or rotorcraft is important, this is not the only factor to design the vehicle. For most cases, performance optimization is first analyzed to fix or regulate the design

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parameters as shown in Figure 1. Therefore, application of ROP to the entire vehicle requires not only weight but also size and other factors to be considered. The resulting process would be very complicated and become impractical.

Maintenance scheduling for a given system is currently performed by two groups, i.e. technicians at the practical field and mathematicians at the academia. The technicians rely on the engineering judgment and experience to set up the maintenance schedule, but mathematicians derive the optimal maintenance policies based on the mathematical theories and numerical optimization. The approach used by technicians is simple, but the subjective decision making may result in sub-optimal or infeasible solution to induce undesirable cost. The analytical approaches to solve for the optimal maintenance policy in academia seem very robust and reasonable. Nevertheless, the analytical approaches are usually accompanied by oversimplification or many specialized assumptions to increase the mathematical tractability. Assumptions in the mathematical models and limitations will be addressed in Chapter 2.

The discussion above generates the motivation to construct the methodology which can be applied to a complex system, such as applications used in Aerospace Engineering, while minimizing inputs from the subjective decision making processes. Furthermore, the methodology should result in the optimal maintenance policy for a given system without any unrealistic assumptions or simplifications.

1.2   Research Goal

As the thesis title indicates, the research is to find the optimal maintenance policies for a system which is consisted of multiple components. The multi-unit system tends to have correlated lifetime distributions and economic dependencies among

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components. As mentioned above, none of the mathematical models can give accurate results for such complex system without imposing any assumptions, and practitioners tend to ignore complexity of the system and perform maintenance scheduling by past empirical data. To narrow the gap between theories and practical applications in constructing maintenance policies for a complex system, aircraft industries have been applying Reliability Centered Maintenance (RCM) to analyze failure modes and to find the efficient maintenance scheduling for a given system [81]. RCM initially defines the problem in qualitative manners (e.g., data analysis and failure effect analysis). As more information is gained, RCM starts to apply some mathematical models to increase the quantitative decision making. Therefore, RCM can be viewed as the hybrid methodology of pure industrial and mathematical approaches.

The proposed methodology from this dissertation also achieves the same end-goal as RCM. The difference from RCM is that the methodology sets up the problem by the quantitative modeling and tries to solve the mathematical models by qualitative ways. The resulting process provides the operators to decide the flexible optimal maintenance schedule for the complex system without any specialized treatments on the system.

1.3 Dissertation Organization

In Chapter 2, existing mathematical approaches to solve for optimal maintenance problem are reviewed. Advantages and limitations of each imperfect model are investigated, and various maintenance policies are considered. Current simulation techniques used in reliability engineering are also examined with advantages and limitations. Based on the advantages and limitations from the literature review, research questions and hypotheses are addressed in Chapter 3, and the proposed methodology is

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introduced in Chapter 4. Information flow block diagrams illustrated in Chapter 4 give the overall view of the proposed methodology. Since the feasibility and implementation of the methodology is not yet proven, Chapter 5 analyzes the seven numerical examples to compare the results from the current available methodologies. Each example lays emphasis on a certain area such as simulation efficiency or mathematical representation of the proposed methodology. It will be concluded from the numerical examples that the proposed methodology has benefits over the current available approach. Chapter 6 implements the proposed methodology to the inspection interval problem of Time Limited Dispatch (TLD) of Full Authority Digital Engine Control (FADEC) system. The simple continuous Markov model used in current available approach is gradually improved by introducing strictly IFR distributions for components and considering dependencies among components. It would be discussed how the proposed methodology is flexible enough to solve the modified system by the help of MC simulation and RSE construction discussed in Chapter 4. Finally, conclusion is shown in Chapter 7 to address the contributions and the further applications.

CHAPTER 2　PAST AND CURRENT EFFORT

The brief introductory concepts on the maintenance models are discussed in this chapter. As early as 1960, researchers began to think about the mathematical representation of the component reliability, and various maintenance models have been introduced to have maintenance policies with minimum cost. Searching for maintenance policies under probabilistic environment became very popular topic in Mathematics, Operational Science and Industrial Engineering. The survey of current work is well organized in [25, 27, 100]. It is found out that the maintenance models rely heavily on the mathematical concepts such as probability theory and stochastic processes, so it is not very widely researched and implemented in other realms of academia. Due to unbalance in the research areas, it will be shown in the following sections that there are limitations in the existing mathematic approaches if these were going to be applied to real practical systems. These limitations provide the starting point for the research questions and improvements. Next sections will address popular maintenance models and simulation techniques used in reliability engineering.

2.1　Maintenance Models

2.1.1　General Ideas of Maintenance Models

Maintenance models can be classified by two major categories: corrective and preventive. Corrective maintenance (CM) is performed when the system or the component fail. Many researchers regard CM as a repair. On the other hand, preventive maintenance (PM) is applied when the system or the component is working [98]. The

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classification mentioned above is based on the maintenance interval. The maintenance can be also categorized by the conditions of the item after the maintenance.

1. Perfect maintenance: It is sometimes called as 'as good as new' that the system is restored to the new condition after the maintenance. Most of introductory reliability theories are based on the perfect maintenance which implies that whenever repair is performed to the failed item, it is repair as new unit. Complete overhaul of an engine with a broken connecting rod can be considered as a perfect maintenance. Usually, a replacement of failed item with new one is in this category [98].

2. Minimal maintenance: Researchers found out that the lifetime of repaired item is not as same as the new one. Minimal maintenance is sometimes called as 'as bad as old'. Barlow and Hunter (1960) first proposed such maintenance that the system operating state is unaffected by the action of maintenance. The resulting survival function for the repaired item is not as same as the new one. It is conditioned on the repaired time, and it can be expressed as equation (1) [3]. It is expected that the residual lifetime would be decreased for the successive repairs. Detail property of minimal repair would be discussed in section 2.1.2.1 with (p, q) policy.

$$S^*(x) = P\left(T \geq x + \tau \mid T \geq \tau\right) = \frac{S(x+\tau)}{S(\tau)} \tag{1}$$

3. Imperfect maintenance: This maintenance lies between the perfect and minimal maintenance. The action of maintenance restores the system operating state to somewhere between 'as good as new' and 'as bad as old'. Imperfect maintenance

model is the major area of research that various policies assume this type of maintenance. The preference of imperfect maintenance is to avoid extreme cases. A perfect repair is ideal case, and a minimal repair is too conservative for most practical applications. The assumption based on the minimal repair is that a minor component is failed, and repairing the component does not affect the system reliability. Sometimes, it also assumes a repair of a wrong (working) item at wrong time [18]. Theses situations are not often observed in real life, so imperfect maintenance became more popular and generally accepted to represent the reality.

There are also other categories that describe maintenance action which makes the system states worse or lead to system failure [69]. Though out this dissertation, most of analysis is performed under the assumption of imperfect maintenance.

2.1.2 Popular Maintenance Models

Various modeling methods have been proposed to represent imperfect maintenance. Each of them has its own assumptions and characteristics that induce advantages and limitations.

2.1.2.1 $\left(p(t), q(t) \right)$ Policy

The policy is widely used through out the literatures from simple set up to represent imperfect maintenance. Perfect maintenance is performed with probability $p(t)$ and minimal maintenance with probability $q(t) = 1 - p(t)$. This binomial setting provides the expected maintenance to be between the perfect and minimal maintenance to represent imperfect maintenance. Brown and Proschan (1983) set the $p(t), q(t)$

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parameters constant to show a distribution of successive perfect maintenance and a corresponding failure rate for a given lifetime of a distribution, F [19].

$$F_p = 1 - (1-F)^p \quad , \quad r_p = pr \qquad (2)$$

Equation (2) gives the simple idea that the binomial setting allows a whole process to become a regenerative process that whenever probability of p happens, the system becomes as good as new [83]. Now we may ask what would happen during the consecutive minimal maintenance with probability of q. It is well discussed and proved by Ascher and Feingold (1984) that under the minimal maintenance, the occurrence of failures follows Non-Homogeneous Poisson Process (NHPP) [3].

$$P\left(N(t_2) - N(t_1) = n\right) = \frac{e^{-\int_{t_1}^{t_2} \lambda(t)dt} \left(\int_{t_1}^{t_2} \lambda(t)dt\right)^n}{n!} \qquad (3)$$
$\lambda(t):$ intensity function

The property of NHPP implies that each lifetime after the failure is neither independent nor identical. The dependency among lifetime distribution after successive repairs is followed by equation (1). It is also observed that, as intensity function increases with time, the system is deteriorating which indicates that there would be more failures occurring for a given time interval.

Leemis (1995) discussed that the intensity function of equation (3) can be substituted by the hazard function of the first lifetime distribution. Moreover, he stated that the expected number of failure by time t under the minimal repair can be modeled as in equation (4).

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$$\Lambda(t) = \int_0^t \lambda(\tau)d\tau = \int_0^t h(\tau)d\tau$$

$h(\tau)$: hazard function

(4)

Other researchers, Block et al. (1985), expended (p, q) to be the function of time. The setting is more practical, since the frequency of a perfect maintenance would be increased to ensure the system safety and reliability as time passes. This is called as $(p(t), q(t))$ modeling, and random variables of parameters slightly modify equation (2) to have a more generalized distribution of successive perfect maintenance and a failure rate [16].

$$F_p = 1 - \exp\left\{\int_0^t p(x)[1 - F(x)]^{-1} F(dx)\right\}$$

$$r_p(t) = p(t)r(t)$$

(5)

It is good time to mention about the advantages of this modeling methodology. It is shown in the equation (6) that it is simple to calculate expected number of failures by time t due to NHPP property, since cumulative hazard function of the first lifetime distribution is only required in the calculation by equation (4). Cumulative hazard function is analytically tractable from probability density distribution or survival function.

$$H(t) = -\log\left[\int_t^\infty f(\tau)d\tau\right] = -\log S(t)$$

(6)

Another advantage is the analytically tractability of distribution of successive perfect CM under $(p(t), q(t))$ modeling. The closed form of distribution can generate the ordinary renewal process to calculate long-term-average metrics such as availability, reward and reliability.

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Besides the advantages mentioned above, there are limitations with $\left(p(t), q(t)\right)$ modeling from the assumptions of minimal maintenance. It is already discussed that the minimal maintenance is too conservative approach for practical applications, and lifetime after the successive minimal maintenances is not intended. For example, the lifetime of push-rod is initially modeled as Weibull distribution. After it fails, minimal CM is performed based on $\left(p(t), q(t)\right)$ policy. Even though the lifetime of push-rod was initially Weibull distribution, it does not guarantee that the distribution after the minimal CM follows Weibull distribution, or any other distributions we know of. The distributions after CM are constructed by the definition of minimal maintenance as in equation (1).

Another limitation is from the ignorance of repair time. NHPP for modeling the successive failure from minimal CM does not consider the repair time. Therefore, the item is repaired immediately, and operating time does not have a discontinuity from the repair. Since there is no repair time, or down time, it is simple to understand that availability is 1 under NHPP assumption for any time interval which makes impractical for real applications.

Finally, there has been little research on the multi-component maintenance under dependencies based on the $\left(p(t), q(t)\right)$ policy [102]. Indeed, this is the problem of the entire maintenance models out there. The closed-form solutions by mathematical derivations for a given maintenance problem are the goal the current mathematical approaches, and the processes require assumptions to guarantee the existence of the closed-form solutions. Moreover, the structure of the system is simplified not to have any dependencies from failures or economic aspects among components to make the mathematical derivations easier. However, it is observed from the practical applications

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that the failure of a certain component may induce other components to fail much faster than before. This is one type of failure dependencies in system [14, 63, 65, 75]. Economic dependency would be discussed in detail at the Chapter 6 with numerical examples.

2.1.2.2 Improvement Factor Model

It is discussed that the minimal maintenance makes the survival function after the CM not to start from 1 but from the previous value when the failure is occurred. Another methodology to represent this condition is by a failure stand point. CM under perfect maintenance can make the failure rate to start from zero. On the other hand, CM under minimal repair results in the failure rate to start from the point where the system has failed. Based on a failure point of view, Malik (1979) first introduced improvement factor model to represent imperfect maintenance. Figure 2 shows how improvement factor method can represent imperfect maintenance [61].
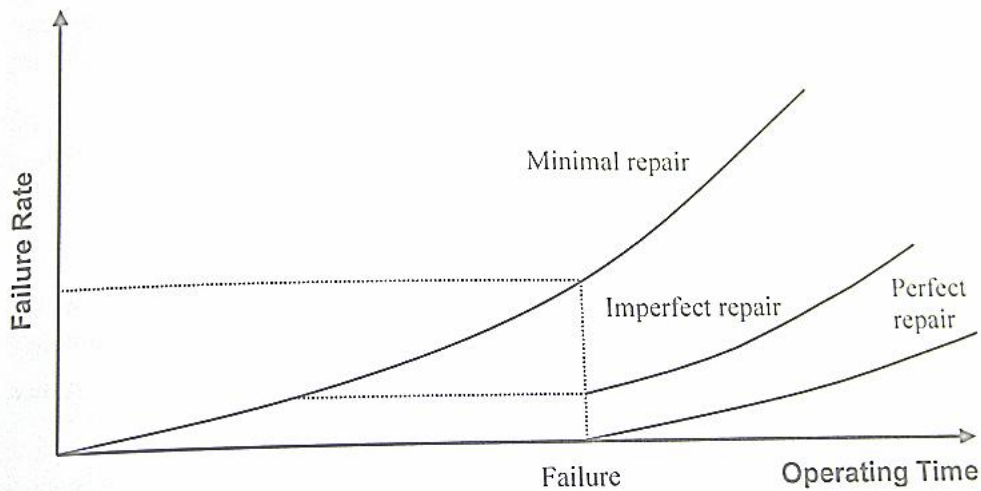
Figure 2: Improvement Factor Method

Imperfect maintenance in the improvement factor model lowers the failure rate than the status when failure is occurred, but it does not lower the failure rate to zero as in

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the perfect maintenance case. The model is extended by Chan and Shaw (1993) to have improvement factor as a function of item's age and the number of maintenances performed [21]. Two types of improvement factor are shown below.

$$r_n^*(t) = r_n(t) - \Delta_n$$
$$\text{Fixed reduction: } \Delta_n = \Delta \qquad\qquad (7)$$
$$\text{Proportion reduction: } \Delta_n = \Delta_{n-1} + g \cdot r_{n-1}(t_{n-1})$$

Following figures show the different types of failure reduction by equation (7).
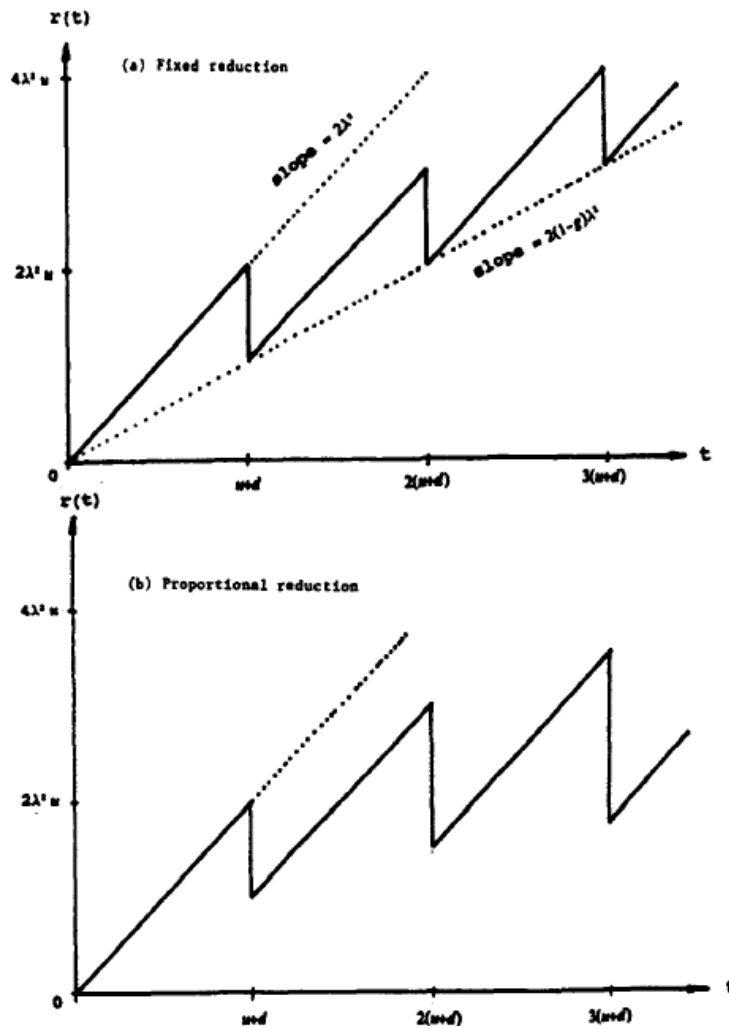
Figure 3: Failure Rate with Different Reduction Types

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The improvement factor method is good for the maintenance policy with failure rate constraint, since the model directly deals with the failure rates. Nevertheless, the methodology also has similar problems as in $\left(p(t), q(t)\right)$ modeling. The resultant lifetime distribution by the failure reduction, i.e. improvement, may not be the distribution that we know of. There may be the case when the resulting lifetime distribution is too complicated to be used for availability or other optimal criteria such as cost rate and reliability. Furthermore, the improvement factor method has not been thoroughly implemented for a multi-unit system or systems with various sources of dependencies.

2.1.2.3 Quasi-Renewal Processes with $\left(\alpha, \beta\right)$

The model assumes expected successive time to failure decreases by fraction of α and expected successive time to repair increases by fraction of β. The mathematical representation is shown below [104].

$$
\begin{aligned}
&\text{Nonnegative random variable } \left\{X_1, X_2, \cdots, X_n\right\} \\
&\text{First interarrival time is } X_1 \text{ then } X_1 = Z_1, X_2 = \alpha Z_2, \cdots, X_n = \alpha^{n-1} Z_n \quad (8)\\
&\text{where } Z_i\text{'s are iid and } \alpha > 0
\end{aligned}
$$

The usual setting for the parameters is that α is lesser or equal to 1 and β is greater or equal to 1 by the equation (8). The properties of quasi-renewal process are shown in equation (9) for probability density function, survival function, hazard function and expected value.

$$
\begin{aligned}
f_n(x) &= \alpha^{1-n} f_1(\alpha^{1-n} x), \quad s_n = s_1(\alpha^{1-n} x) \\
r_n(x) &= \alpha^{1-n} r_1(\alpha^{1-n} x), \quad E(X_n) = \alpha^{n-1} E(X_1)
\end{aligned} \quad (9)
$$

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The hazard rate function after n successive maintenances is analogous to proportional improvement model. Moreover, it also accounts for reduction in lifetime if $\alpha > 1$ to represent the model similar modeling as in $\big(p(t), q(t) \big)$ policy. The term quasi-renewal is used by researchers, because the interarrival distribution is not identical as shown in the equation (9). Instead, it is proven by Wang and Pham (1996b) that quasi-renewal process has an equation for the expected number of failure during interval which is very close to the renewal function of the ordinary renewal process.

$$
\begin{aligned}
M(t) &= E[N(t)] \\
&= \sum_{n=1}^{\infty} P\big(N(t) \geq n\big) \\
&= \sum_{n=1}^{\infty} G^{(n)}(t)
\end{aligned}
\tag{10}
$$

$G^{(n)}(t)$: convolution of interarrival times

It is also proved that the first interarrival distribution of a quasi-renewal process uniquely determines its quasi-renewal function by one-to-one correspondence of distribution and its Laplace transformation. Therefore, the quasi-renewal function can be directly calculated from quasi-renewal equation. This is shown in section 5.3 with numerical examples.

The properties of quasi-renewal process induce advantages of using this model for the imperfect maintenance. The probability density function after the maintenance has same properties of the first lifetime distribution. Wang and Pham (1996b) showed that if the first lifetime distribution follows Weibull, Gamma or Lognormal distribution then the shape parameter of distribution after the successive maintenances is preserved. A Shape parameter of a lifetime of a hardware product tends to relate to its failure mechanism and

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modes [104]. For example, if we start with Weibull distribution with scale parameter λ and shape parameter κ then the resulting scale and shape parameters would be $\lambda_n = \alpha^{1-n}\lambda$, $\kappa_n = \kappa$ after $n-1^{th}$ maintenance. As a result, expected value of lifetime can be calculated, and this is consistent with equation (9).

$$
\begin{aligned}
E[T_1] &= \mu_1 = \frac{1}{\lambda\kappa}\Gamma\left(\frac{1}{\kappa}\right) \\
E[T_n] &= \mu_n = \frac{1}{\alpha^{1-n}\lambda\kappa}\Gamma\left(\frac{1}{\kappa}\right) = \alpha^{n-1}E[T_1]
\end{aligned}
\tag{11}
$$

Ascher and Feingold (1984) gathered the interarrival lifetime data from the successive maintenances of bus engines. The bus engine lifetimes were recorded in mileage between failures, and it is observed that the mileage between failures decreased as the number of CM increased [3]. The data set can be used for the statistical hypothesis test to reject or accept the null hypothesis that there is a common proportion, α, which is applied through the reduction in successive lifetimes. It is not the scope of this dissertation to determine the proper α value for a given data. The bus engines lifetime data suggest that obtaining the quasi-renewal parameters is the statistical problem which indicates that if there are not many evidences to reject the null hypothesis then it is statistically significant to use quasi-renewal parameters to represent the real applications. Unlike minimal maintenance model, which starts the problem with the assumption of model, Quasi-renewal process provides operators to decide the right parameters for the model by statistical estimation.

Besides the statistical testability, quasi-renewal model has a limitation in calculating quasi-renewal function, i.e. expected number of failures. Generally, it is not

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easy and sometimes impossible to directly solve (quasi-) renewal equation for Increasing Failure Rate (IFR) distributions. Direct mathematical approach generally utilizes Laplace-Stieltjes transformation, but most IFR distributions can not have the closed functional form of Laplace transform or the inverse Laplace transform of the renewal function. The example in section 5.3 deals with a special case by Gamma of order 2 to make direct approach possible. That is why there are methodologies to approximate the renewal equation with infinite series [58, 93].

Quasi-renewal model also has a limitation from implementing it to the multi-component maintenance problems. Wang and Pham (2006) discuss the parallel system under correlated failure and repair under quasi-renewal process [101]. They assumed constant failure rate for the lifetime distribution to satisfy the convergence criteria. It can be concluded that the recent paper under quasi-renewal processes also has assumptions from mathematical derivations. It is also simple to find that most authors set up special assumptions on the repair time to reduce the complexity in their applications.

2.1.3 Popular Maintenance Policies

Based on the various imperfect maintenance models addressed in the previous section, different types of maintenance policies have been developed. The numerical optimization problem is constructed under various maintenance policies to reach the optimal maintenance. The optimization problem generally has the long-run average cost rate as an objective function and the long-run average availability as a constraint. The term 'long-run average' is to have the steady state property of a given metric. All the values in optimal maintenance problem are the steady state values, so if term 'long-run average' is abbreviated in following sections, it still implies the steady state properties.

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Cost rate is used more often than total cost, since the total maintenance cost can be misleading by the time horizon. The mathematical definition of cost rate can be structured as renewal reward process, if there is a renewal cycle that make the whole process to repeat itself. The steady state value of cost rate is same as the cost rate up to system renewal cycle by key renewal theory [83] as in equation (12).

$$L(t) = \lim_{t \to \infty} \frac{C(t)}{t} = \frac{C(D)}{D}$$

$C(t)$: maintenance cost up to time t

$C(D)$: maintenance cost up to time D (12)

D : system renewal cycle

Long-run average availability is constructed similar way as the long-run average cost rate by use of system renewal cycle to have renewal process.

2.1.3.1 Age Dependent Maintenance Policy [5, 6]

This policy addresses that maintenance is performed if failure happens or the item reaches certain age T, whichever occurs first. This section examines the perfect maintenance, i.e. replacement, case for the mathematical derivation. The imperfect maintenance can be extended by $\left(p(t), q(t) \right)$ policies as mention in previous section [10].

$$
\begin{aligned}
L(t) &= \lim_{t \to \infty} \left[c_1 \frac{E[N_1(t)]}{t} + c_2 \frac{E[N_2(t)]}{t} \right] \\
&= \frac{c_1 F(T) + c_2 S(T)}{\int_0^T \tau \, dF(\tau) + T \int_T^\infty dF(\tau)} \\
&= \frac{c_1 F(T) + c_2 S(T)}{\int_0^T S(\tau) d\tau}
\end{aligned}
$$

(13)

c_1 indicates the cost of maintenance at the failure before reaching the age T, and

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c_2 is the maintenance cost when the system reaches age T. From this setting, overall maintenance cost would be the sum of all the c_1 and c_2 during the interval $[0,t]$. The expected number of failure is written as $E[N_i(t)]$, $i=1,2$. By key renewal theory, the limiting value is same as only considering the values up to the renewal cycle. The renewal cycle ends whenever the first failure or age of T happens. This is shown in the second line of equation (13). It can be proved that denominators of second line and third line are equivalent by Leibnitz's theorem of calculus [48]. The optimal condition for the age dependent maintenance policy can be obtained by setting the first derivative of $L(t)$ equal to 0.

$$r(T^*)\int_0^{T^*} S(\tau)d\tau - F(T^*) = \frac{c_2}{c_1 - c_2} \tag{14}$$

The relation between c_1 and c_2 will influence over the existence of the optimal solution. If there is no solution or $x^* = \infty$ by running the optimization then the optimal policy would be system maintenance by CM only.

2.1.3.2 Periodic Maintenance Policy

Periodic maintenance is performed at fixed time interval. A component receives (imperfect) PM at every T time unit. The mathematical representation for the perfect maintenance is shown in equation (15). This model is called as Block replacement policy which would be discussed with the numerical example in section 5.3.

$$L(T) = \frac{c_1 M(T) + c_2}{T} \tag{15}$$

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Key renewal theorem is also applied in equation (15) that periodic maintenance interval general the renewal cycle. The optimal condition is shown in equation (16).

$$T^* m(T^*) - M(T^*) = \frac{c_2}{c_1} \qquad (16)$$

The existence of solution and the properties are analyzed in section 5.3.

2.1.3.3 Failure Limit Policy

The policy addresses that PM only occurs when the failure rate reaches a predetermined level [14]. This policy is usually accompanied by the improvement factor model, since the model directly deals with failure rate as mentioned in section 2.1.2.2.

2.1.3.4 Sequential Maintenance Policy

If the system is maintained at unequal intervals, the policy is called as sequential PM policy. The unequal interval can be related with the age of the system or the predetermined interval like in periodic maintenance policy. Barlow and Proschan (1962) compared sequential maintenance policy with age dependent policy and concluded that the flexibility in sequential PM policy induced the lower cost rate [4]. Nakagawa (1986, 1988) implemented sequential PM with improvement factor model [67, 68]. General procedure of solving for optimal maintenance policies under the sequential PM policy begins by dividing predetermined interval into the number of successive time and generating cost rate function with successive maintenance time. Then, the necessary condition is derived by the first derivative of objective function with respect to each successive time interval. In most cases, minimal CM is assumed, since it gives very simple relationship for the expected number of failure during the interval by equation (5).

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System of equations of optimality condition is solved to have sequential maintenance policy. If quasi-renewal process is used for imperfect maintenance then it has a same problem from intractable renewal function as mentioned in section 2.1.2.3. That is the reason why Wu and Clements-Croome (2005) proposed assumptions on the renewal cycle to simplify the decision making under the sequential maintenance policy [107].

2.2   Simulation Methodologies

Generally, Monte Carlo simulation is implemented to get the reliability measures, (e.g., expected number of failures, system renewal cycle, etc.). Monte Carlo simulation is the numerical technique which utilizes probability distributions and random number generators to solve complex integration, optimization and so on. The reliability of each component is based on probability distributions, so the flow of system can be modeled by a set of random variables [2]. Therefore, direct Monte Carlo method simulates reliability of item by calculating a number of successes over a number of trials. It is easy to see that the accuracy of simulation depends on the total number of runs in direct simulation. Shreider (1960) proved that the error between simulated result (S) and actual result (A) has following relationship with total number of runs (n) [90].

$$ error = |S - A| < \frac{1}{\sqrt{n}} \tag{17} $$

Equation (17) states that it becomes less efficient, if n is already large enough which indicates that much larger runs are required to improve the accuracy. Following section briefly discusses about Monte Carlo methods used in reliability engineering.

2.2.1   General Ideas of Monte Carlo Algorithm Used in Reliability Engineering

Most available methods are based on the binomial setting which assumes that if the lifetime has passed predetermined time (t) then the component is working, otherwise it is failed. Counting the number of successes and total number of trials simply generates reliability of a system given time (t). The simulation usually requires minimal cut set and path set in advance to calculate the system reliability. The basic algorithm mentioned

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above is called as K-R method by Karmat and Riley (1975) [47]. Rice and Moore (1983) proposed the extension of K-R method by using the normal approximation to the binomial distribution [82]. It is mentioned that number of success (f_i^*) over number of trial (n_i) simulates the probability of the component working given time period (p_i). This binomial setting (B) can be approximated by the normal (N), if sample size is sufficiently large.

$$p_i = \frac{f_i^*}{n_i}, \ q_i = 1 - q_i, \ \text{var}[B] = \frac{p_i q_i}{n_i}$$
$$B \sim N\left(p_i, \frac{p_i q_i}{n_i}\right)$$

(18)

Other researchers, Chao and Huang (1987), proposed the methodology using Bayes theorem to use the information of prior distribution over the posterior distribution [23]. Binomial distribution of success or fail can be represented by beta distribution to add more information as the number of simulation increases. It is important to note that R-M method and C-H method are only valid for the lifetime having binomial distribution.

So far, we have considered the Monte Carlo method for the non-repairable component whose reliability can represent the system status. If the system is repairable as in most aerospace applications then the availability becomes the representation of the system status during a certain time interval. Especially, long-run-average availability gives the good representation of the system status because it has a steady state property.

$$MTBF \approx \frac{t}{E\big[N(t)\big]}$$

(19)

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Quiet a few researches have done on this area. Kamat and Franzmeir (1976) and Kim et al. (1992) presented Mean Time Between Failure (MTBF) estimation based on equation (19) [46, 52]. Kim et al. mentioned in the paper that the estimation, indeed, converged to the actual solution, if simulation number and the time interval are sufficiently large. This result is consistent with elementary theory of renewal which states that, as time t in the equation (19) goes infinite, the right hand side of equation converges to the expected value of renewal cycle, i.e. expected value of lifetime (MTBF). The survey of simulation used in reliability engineering is well discussed in [103].

2.2.2 Other Simulation Techniques for Improvement

The direct Monte Carlo simulation can only reduce the variation of the result by increasing number of trials as in equation (17). There are many methodologies developed to decrease the variation of the simulated result. Nelson (1987) summarized the commonly-used Variance Reduction Techniques (VRT) [70]. Commonly-used VRTs are antithetic variates, control variates, important sampling and so on. If the distribution has monotonicity then introducing a pair of samples can reduce the variation as in antithetic variates method. Exponential distribution is a good example for antithetic variates. Control variates method generates certain random variable which has same expected value but lower variance than the original one. Important sampling is performed by the change in probability measure to increase the accuracy of rare event simulation. There are other methodologies used in reliability simulation. Kumamoto et al. (1987) discussed a sampling technique to exploit the negative correlation by dagger-sampling method, and Chang et al. (2001) utilized complex mathematics from Variational Principle to reduce the variance of the simulated result [22, 56].

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2.3   Summary of Methodologies Used in Reliability Engineering

Previous section discusses different types of models for imperfect maintenance and maintenance policy. Moreover, various simulation methods used in reliability engineering are discussed. It is good time to summarize the models by addressing the advantages and limitations. Examining the properties of each model would lead the research questions which will be addressed in next chapter.

It is important to note that various maintenance policies are not selected from their efficiencies and properties. The selection for the maintenance policy is governed by the system of interest. For example, it is not practical to do the sequential maintenance for the satellite maintenance, since it would be very expensive. Therefore, imperfect maintenance models are compared.

Table 1: Summary of imperfect maintenance models

Models	Advantages	Limitations
$(p(t), q(t))$	Use of NHPP properties Closed form of $F_p(t)$	Discrepancy of $F_1(t)$ and $F_i(t), i > 1$ Neglected repair time Single component analysis
Improvement	Direct relationship with $r(t)$	Less focus on property of $F(t)$ Neglected repair time Single component analysis
(α, β)	Ability of model validation Shape parameter is reserved	Hard to get $E[N(t)]$ Neglected repair time Mostly single component analysis

It is observed from Table 1 that all of the popular imperfect models fail to solve for multi-unit system maintenance problems. Typical multi-unit system consists of multi-

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component with different lifetime distributions and the dependencies among component. These settings make existing mathematical approaches hard to solve such problems under imperfect maintenance models. Repair time is neglected or accompanied by specialize assumptions in all models to reduce the complexity in mathematical approach. If repair time is considered then the convolution of lifetime and repair time would be involved in the renewal cycle. The resulting distribution of successive maintenance is generally not simple to handle, if number of maintenance increases.

Figure 4 illustrates the general classification of maintenance strategies [62], and each box represents the cluster of literatures for given maintenance strategies.
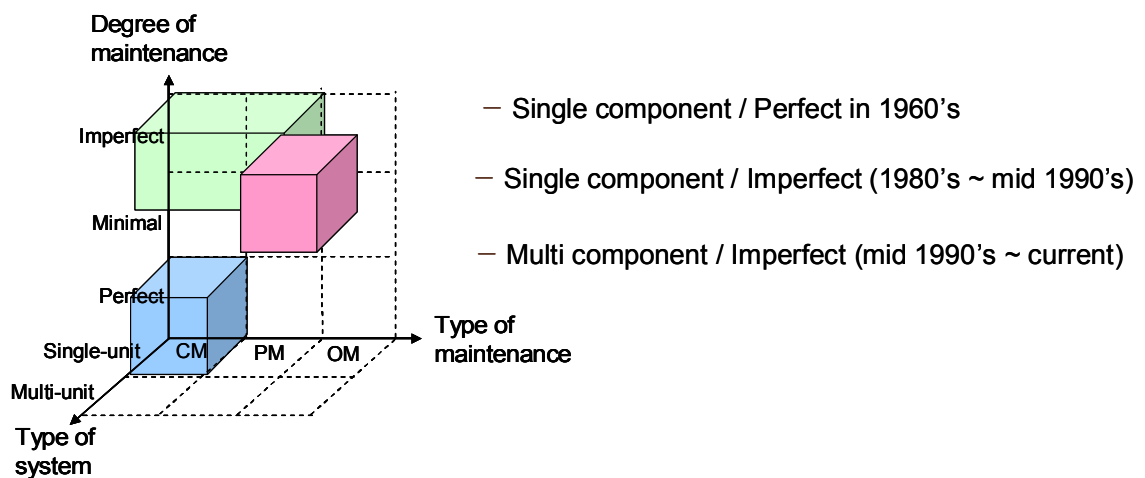
Figure 4: Classification of Maintenance Strategies

The history of literatures on maintenance suggests that the mathematical model becomes complicated as it moves away from the origin. The first maintenance theories were established under perfect maintenance for the single component. Then, the concept of minimal repair was proposed. The imperfect maintenance model has been popular between 1980's and mid 1990's. It is mentioned that most imperfect maintenance models were based on the single component to eliminate the complexity which is required for

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most multi-component settings. The real-world practical applications tend to consist of multiple components, and it is the research objective of this dissertation to set up the maintenance schedule for a multi-unit component, whether the model requires considering any dependencies or not. Following table lists the limitations of the current availability mathematical approaches applied to multiple components maintenance problems.

Table 2: Limitations from Current Mathematical Model for Multi-unit Systems

	Description	Limitations
Zheng and Fard (1992)	Opportunistic maintenance by hazard rate	Neglected repair time
Sheu (1992)	Maintenance under minimal repair	Neglected minimal repair time
Zhao (1994)	Series system availability for general distribution	Constant failure rate
Zheng (1995)	Opportunistic maintenance	Neglected replacement time
Wang and Pham (2000)	Optimal (τ, T) policy for k-out-of-n system	Neglected minimal repair time
Wang et.al. (2001)	Preparedness maintenance under economic dependence	Constant failure rate One component with strictly IFR
Wang and Pham (2006)	Series system availability under correlation	Constant failure rate

Table 2 suggests that the recent articles on multi-unit system maintenance have an assumption on the lifetime or the structure of the system to reduce the complexity in mathematical derivation. There is a trade off between the depth of imperfect model and the dependencies among the components, so it is not simple to achieve two areas at the

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same time, since the resulting approach would not practical to be analyzed under mathematical theories. The limitations from the current available mathematical approaches open the room for the improvements.

Following table summarize the simulation methodologies discussed in the previous section with advantages and limitations.

Table 3: Summary of Monte Carlo simulation methods

Method	Advantages	Limitations
K-R	Direct simulation Availability for any distribution	Considerable large variation MTBF is not considered
R-M / C-H	Usage of prior information	Less variation than direct method Limited to Binomial distribution MTBF is not considered
Kim et al.	Available for MTBF	Neglected repair time Suspended animation

Table 3 shows that all of the methods are not capable of obtaining accurate MTBF. Kim et al. method may give acceptable MTBF value, but the model does not consider the repair time. MTBF is an important measure to define the availability which is used as a constraint in the optimal maintenance problem. Moreover, most MC simulation algorithm for multi-unit system assume suspended animation which implies that component is repair at system failure, and the rest of components do not gain any ages during the repair of the component. There are various shut-off rules to represent suspended animation, and Khalil (1985) compared the results from different rules [51]. Therefore, it is recommended to have flexibility on suspended animation based on the customer requirement.

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CHAPTER 3   RESEARCH TOPIC

It is observed in the previous chapter that there are limitations in current mathematical approaches if they were to be applied to real-world systems. Such systems may have multiple components with dependencies among components It is also mentioned that repair time is neglected in most existing mathematical models to alleviate mathematical complexity. This chapter generates the questions which are arisen from limitations. Then, hypotheses are proposed to improve the current available models.

3.1   Research Question 1: How to Construct the Optimal Maintenance Problem without Mathematical Derivation?

The optimal maintenance problem can be regarded as the constrained optimization problem to solve for the optimal maintenance policies. This optimization problem has a long-run cost rate as an objective function and a long-run average availability as a constraint function. It is mentioned in section 2.3 that one cannot easily construct the closed-form equations for either objective function or constraints of optimal maintenance problem without imposing any specialized assumptions or simplification. These assumptions and simplification enable mathematical derivations of reliability measures (e.g., expected number of failure, system renewal cycle, expected system up time, etc.) to construct the optimal maintenance problem. Therefore, the research question is how to find an alternative approach to construct the objective function and the constraint of the optimal maintenance problem without mathematical derivations.

It is observed that simulation methods have powerful advantages over multi-

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component setting. Even the basic K-R method is applicable for the multi-unit system reliability simulation, and minor modifications on the algorithm enable to include dependencies among components. The nature of simulation does not require users to have any assumptions on the distributions or simplification on the system structure. The arbitrary lifetime distribution can be generated by MC simulation, and the expected number of failure of a component can be obtained by the simulation techniques for MTBF as discussed in Section 2.2. Moreover, minimum cut set or path set allows users to simulate expected system up time for given renewal cycle in order to get the information on availability. For example, if there are two decision variables, i.e., periodic PM interval and periodic inspection interval, then the MC simulation would offer the corresponding value of reliability measure based on the combination of decision variables. If existing mathematical approaches are to be applied to the problem, then mathematical derivations are required to have closed-form formula for each reliability measure with respect to decision variables. There can be no denying that the mathematical derivations may not successfully achieve the goal of having closed-form formula even with various assumptions. Therefore, MC simulation provides the simulation results for reliability measures for given combination of decision variables without any special treatments on the problem.

Based on the advantage of MC simulation, one can search for the optimal combination of decision variables, which result in a minimum cost rate. This technique is defined as coupled optimization in this thesis, and the term implies that numerical optimization is coupled with MC simulation to yield the functional values of both objective function and constraint. For instance, the renewal cycle used as the denominator

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of the cost rate is hard to have the closed-form formula from mathematical derivation for a certain system. MC simulation, however, allows to have renewal cycle value for given decision variables, since MC simulation only requires to generate random distributions to check for the renewal cycle. The coupled optimization make possible to find the optimal maintenance polices for the problem while existing mathematical approaches fail to solve it. Nonetheless, the coupled optimization has a shortcoming from the nature of MC simulation. The simulation result is not fixed value which implies that it will give slightly different answer due to the variation of simulation. Even with the VRT or other techniques, the simulated result will have upper and lower bound. If the search point is far away from the optimal solution then the couple optimization would find the direction to minimize the cost rate. There is, however, a problem when the search point is close to the optimal point. The difference between upper and lower bound of simulation result would be larger than the optimal criteria to stop the numerical optimization. If this happens then the numerical optimization stops at the sub-optimal points, or the optimization to would take a lot of time to have the converged solution.

Another problem can arise from couple optimization. The MC simulation results for reliability measures are the discrete simulation. In contrast, mathematical derivations provide the continuous formulas for reliability measures with respect to decision variables. The continuous formulas have reusability for optimization under different maintenance cost settings (e.g., system PM cost, component CM cost, OM cost, etc.). The MC simulation in couple optimization does not have reusability, and it is required to repeat MC simulation step for different maintenance cost settings. Furthermore, MC simulation generally requires computational time to have a required accuracy, so the

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couple optimization would be inefficient to solve the optimal maintenance problems for various maintenance cost settings.

In summary, simulation can help determine unknown metrics (i.e., reliability measures) involved in the objective function or constraints of the optimal maintenance problem, since it does not require any assumptions or simplification on the system as in derivation processes used in existing mathematical approaches. Nevertheless, it is mentioned that the direct MC simulation with numerical optimization, which is called as coupled optimization, is vulnerable to have sub-optimal solution with very slow convergence. Moreover, having continuous formulas for reliability measures is required to increase the reusability in order to shorten the process time.

3.2   Hypothesis for the Research Question 1

If one can extract parametric formula from the simulated result then the research question can be answered. The approach not only uses the benefit from the MC simulation, but also has reusability from continuous formulas of reliability measures.

If there is only one decision variable involved with the simulation process then the parametric formula can be obtained from curve fitting. For instance, periodic optimal maintenance policy has one variable, i.e. PM interval, to be determined. The expected number of failure and the availability are the function of PM interval. It would be very nice to have closed-form formula on the metrics by mathematical derivations, but previous section mentioned that it is usually very hard or impossible for a multi-unit system. MC simulation is applied to gather information on these metrics. The scatter simulation results for reliability measures are obtained from MC simulation by varying PM interval. Then, the parametric formula with respect to PM interval is constructed to explain the simulation data as much as possible. In this example, curve fitting can be used to generate the parametric formula. Maximum Likelihood Estimator (MLE) can be implemented to construct the parameters of the formula [29, 59]. The simple example is shown in Figure 5 for the Gamma with $\lambda = 0.05$ and $\kappa = 2$. Laplace Stieltjes transformation and algebraic manipulation can yield the analytical solution of $M(T) = 0.25\,\mathrm{e}^{-0.1T} + 0.025T - 0.25$ which is shown with the blue curve. The red curve illustrates the curve fit result of $y = -0.000008T^3 + 0.000718T^2 + 0.003534T - 0.007527$ from curve fit to extract parametric formula from MC simulation.
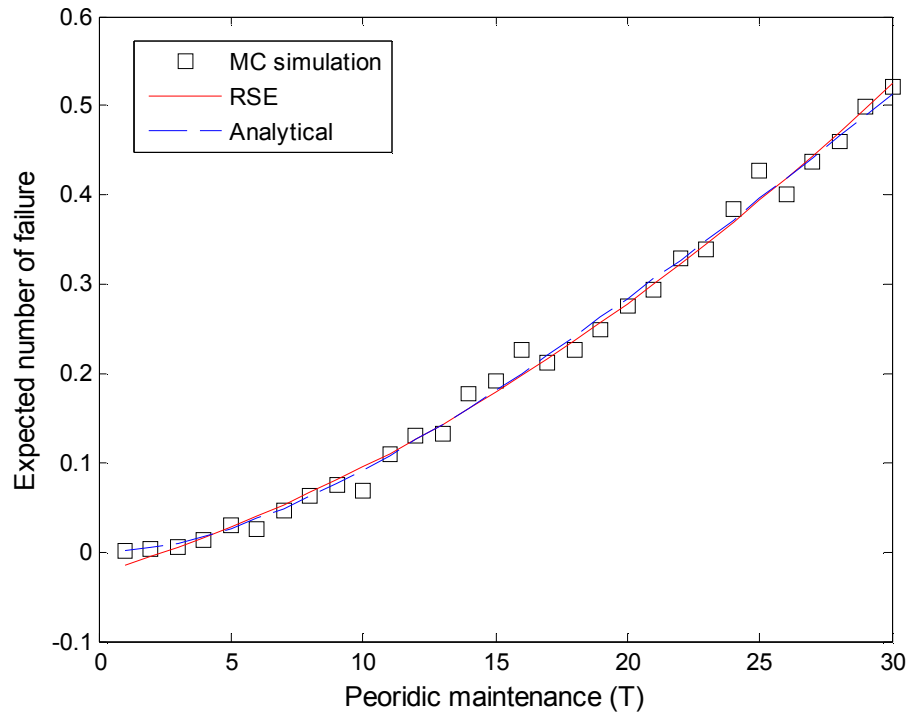
Figure 5: M(t) curve fit for Gamma lifetime distribution

The above example shows how to extract formula from MC simulation. The expected number of failure is required to construct the cost rate in the objective function. Moreover, simple polynomial equation from curve fit result for this measure can be used instead of the analytical result, which is not guaranteed to be obtained by mathematical derivation for most cases. The simpler equation from curve fit also helps the numerical optimization, since the cost rate is already non-linear function, and it is better to have a simpler equation to reduce the computational time.

The similar procedure can be applied to the case when there are more than 2 decision variables in the optimal maintenance problem. The curve fit, however, would not be very applicable for this case, since MLE for multi-dimensional problem is not practical, and interaction terms among the decision variables in parametric formula make

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the estimation process much harder to be implemented. The multi-parameter regression can be done by Response Surface Methodology (RSM). RSM utilizes Response Surface Equation (RSE) to approximate the inherent dependence of functional response to a series of design variables using a lease-square regression [53]. To implement RSM for the multi-parameter regression, first, different combinations of decision variables are required for the MC simulation. These combinations are later being denoted as Design of Experiments (DOE) table, and MC simulation is performed as in the single variable case to gather the information on the corresponding reliability measures. Then, simulation results are regressed by RSEs which are initially constructed as quadratic functions based on the decision variables. The least square fit method for regression provides the proper estimate of each parameter to have the representative formulas for reliability measures. More detail implementation of RSE is discussed in Chapter 4.

3.3 Research Question 2: How to Improve the Accuracy of the Simulation Technique?

The usage of RSM seems applicable to construct continuous functions for reliability measures without having intensive mathematical derivation processes. Nevertheless, the approach relies too much on the MC simulation results. If MC simulation is wrong or misleading at the first hand, there is no way to have correct RSEs for reliability measures. The current available MC simulation techniques neglect the repair time. Moreover, Perfect repair under suspended animation assumption also overestimates system availability as discussed in previous chapter. These assumptions on repair result in inaccurate simulated results which should be prevented.

Besides the component level, the dependencies among components are not considered in existing simulation algorithms. The dependencies among components tend to change the lifetime distribution of the component based on the failure of other components in most real-world applications. Therefore, considering dependencies in simulation is important to have accurate responses on metrics such as expected number of failure, expected system up time.

It is true that there are few algorithms which try to address the limitations mentioned above. The state-of-art simulation algorithm in reliability engineering is review in Section 5.1 in detail with the proposed methodology. For now, the improved MC simulation is required in order to increase the accuracy to represent the real-world applications.

3.4   *Hypothesis for the Research Question 2*

To increase the simulation accuracy from current MC algorithms can be simply done by including more features in simulation. If the repair time is considered to be important then include in the simulation. If the operators think suspended animation may give overestimated result then relax the assumption by recording the ages of other components, while repairing one component. One may wonder why researchers kept assuming the suspended animation for long time. Suspended animation assumption increases analytical tractability, and mathematicians want to complexity to be alleviated to reach the final closed-form solution. If suspended animation is assumed then other components, which were suspended during the maintenance, only gains the constant time after the maintenance. Indeed, under suspended animation, it requires one line of mathematical equation to represent the state of each component after the maintenance of one component by simple addition of constant (i.e., expected repair time) in each distribution parameters [102]. If the assumption is to be relaxed it would involve probabilities to account for any random time of failure that may occur during the maintenance. On the other hand, if concurrent simulation (see Chapter 4 for the notation) is applied to relax the suspended animation assumption then the aging occurred in other components will change the probability of failure during the repair of another one.

Imperfect maintenance can be model by quasi-renewal process, since it preserves the shape parameter for the successive distributions, if either of Weibull, Gamma and lognormal is used. The further description is mentioned in following chapter.

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CHAPTER 4 PROPOSED METHODOLOGY

Based on the hypotheses of the research questions from the previous chapter, new methodology is proposed with information flow diagram, and the general description on the methodology is discussed throughout this chapter. More detail procedure for different modules in the proposed methodology would be examined with numerical examples in following chapter. Flow diagram of a current available mathematical approach and proposed methodology are compared to readdress the differences and improvements.

4.1   Flow Diagram of the Current Available Mathematical Approach
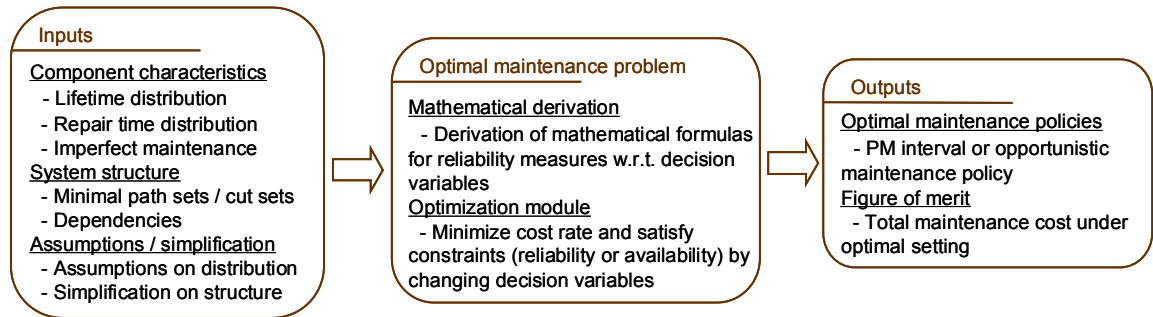
Figure 6: Flow Diagram of the Current Available Mathematical Approach

As mentioned in Chapter 2, all the inputs are given to the operators at the first hand. Lifetimes of most mechanical or electronic components tend to follow IFR distributions to avoid reaching a trivial solution of infinite PM interval as in Decreasing Failure Rate (DFR) distributions. System structure can be obtained from Reliability Block Diagram (RBD) or transition diagram to represent the system of interest. Distribution parameters and system structure are given to the operators by the real data on components and physical arrangement of the system. On the other hand, imperfect

maintenance in the component characteristics is one of the design factors which operators should consider to have more realistic maintenance. It is observed that the various imperfect models in Chapter 2 have influence on the distribution parameters.

The assumptions and simplifications in input are required for the mathematical derivation in the middle block, where reliability measures, such as expected number of failure of a component or renewal cycle, are derived to construct the objective function and constraints of the optimization problem. Generally, mathematical derivation requires knowledge from probability theories and stochastic processes. Nevertheless, the derivation task can quickly become unmanageable when the system under scrutiny happens to be complex, with multiple dependencies to failure, economics, etc. It explains why most state-of-art mathematical approaches enable measures to increase the mathematical tractability of the optimization at hand. For example, assumptions are allowed to be made about the properties of the input probability distributions which can serve to simplify the derivation process. In other instances, simplification is applied to the entirety of the system's structure in an attempt to reduce the failure dependencies.

After setting up the problem by mathematical derivation, the numerical optimization is solved to result in the optimal maintenance policies. Maintenance policies may include what is found to be the optimal periodic PM interval. Opportunistic maintenance scheduling can also be a part of the policies, if economic dependency is desired to be considered. Resulting total maintenance cost or availability of the system under optimal maintenance policies may be considered as figure of merit.

4.2 *Flow Diagram of the Proposed Methodology*
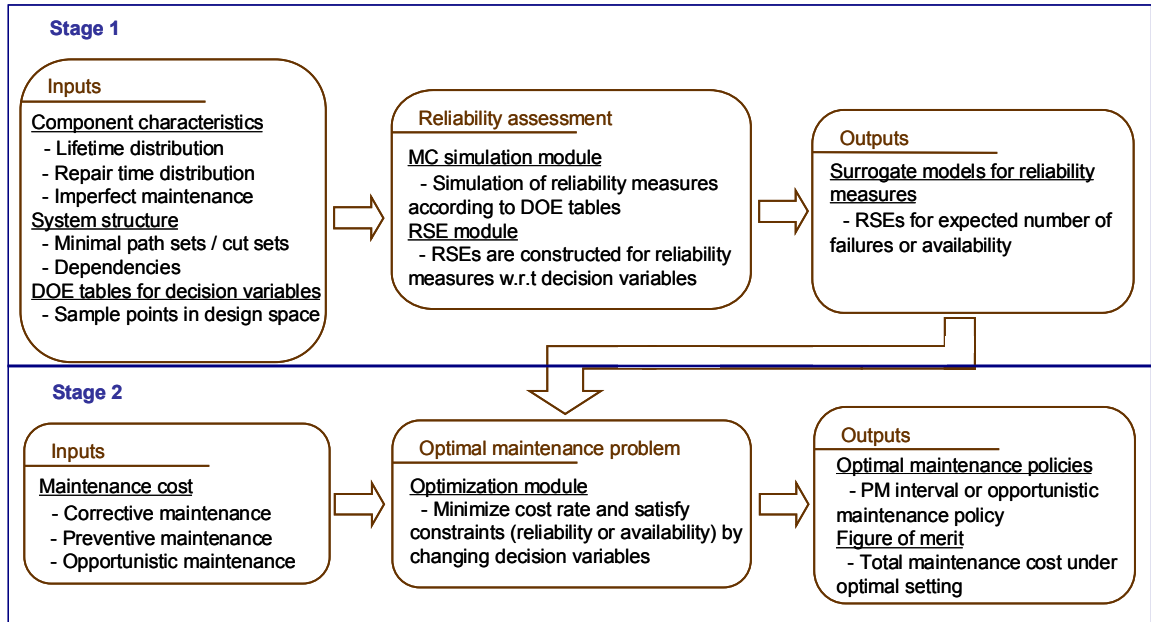
Figure 7: Flow Diagram of the Proposed Methodology

The main difference from the current available mathematical approach is that the proposed methodology does not require mathematical derivation. Instead, it utilizes MC simulation and RSEs to construct surrogate models for the unknown functions in the objective function and constraints of the optimal maintenance problem. It is reviewed in Chapter 3 that MC simulation does not require any assumptions or simplification on distributions or dependencies among components. Moreover, RSEs can serve good representations over the discrete simulated data sets. These RSEs replace reliability measures to reconstruct the objective function and constraints, and numerical optimization is performed to result in the optimal maintenance policies. Following diagram shows the comparison between mathematical approach and the proposed methodology.

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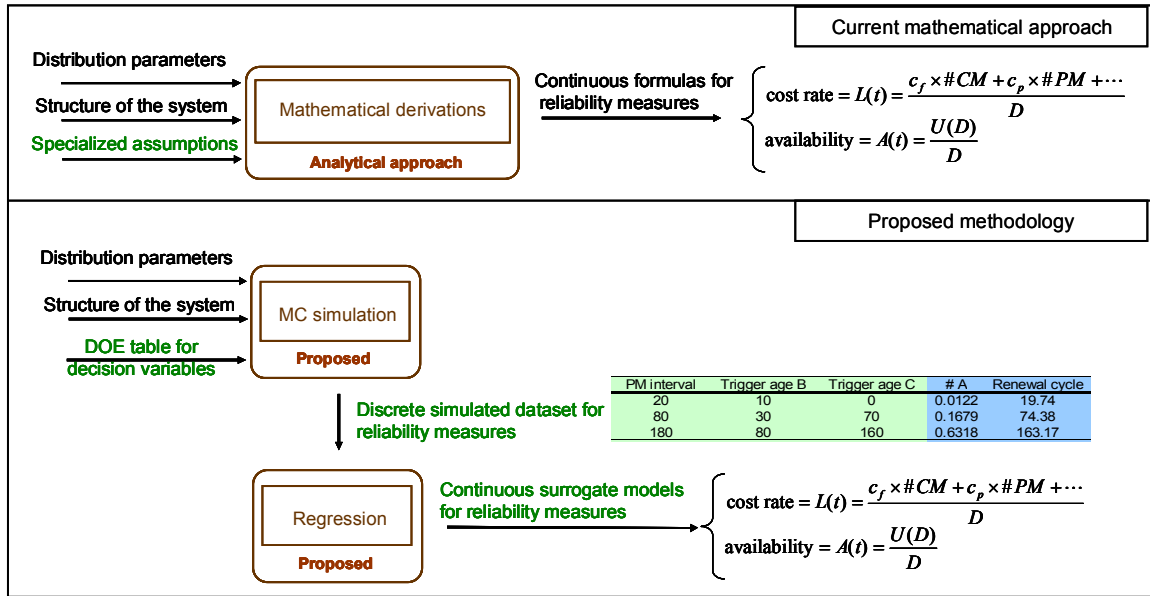
Figure 8: Comparison between Mathematical Approach and Proposed Methodology

The mathematical derivations in the current available methodology requires specialized assumptions on distributions or the structure of the system as mentioned in Chapter 2. The proposed methodology utilizes MC simulation and regression process to eliminate mathematical derivations to construct continuous formulas for reliability measures. The MC simulation provides discrete simulated data sets as shown in Figure 8. The three variables in the green box are the decision variables, and the combination of the decision variables are set by the given DOE table. Each row of DOE table is simulated through the MC simulation to result in corresponding values of reliability measures in the blue box. These discrete data are regressed to have continuous surrogate models. The sample RSEs for this problem can be found in section 5.7.

The following sections focus on the each module in the overall flow diagram of the proposed methodology.

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4.3 MC Simulation module
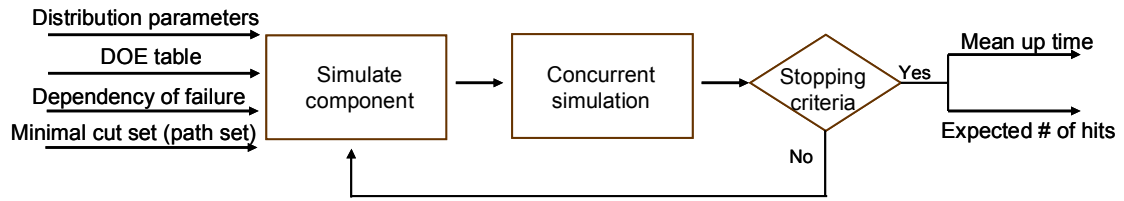
Figure 9: Flow Diagram of Simulation Module

The simulation module has iterative three steps. During the 'simulate component' step, each distribution is simulated by the inverse transformation or well known algorithm. 'Distribution parameters' in input are available by the data sources. If the component is standardized and purchased from the industry, the lifetime distribution can be obtained from the experimental data. Quasi-renewal process is modeled to represent the imperfect maintenance, and it is assumed that proper factors are given at the first hand by statistical test. The usual setting for quasi-renewal parameters to describe the chances of imperfect maintenance is $\alpha = 0.95, \beta = 1.05$. It can be understood that 5% margin is used to incorporate change in distribution parameters from the imperfect maintenance. The second input of Design of Experiments (DOE) table is used to setup the sample points for discrete simulation to obtain information (behavior) of the reliability measures in constructing the objective function or constraints of the optimal maintenance problem. Typically, the DOE tables for constructing a second order polynomial RSE are 3-level designs which includes Central Composite Design (CCD) or Box-Behnken Design [17, 66, 94]. If the out-of-sample error is unacceptably high for the problem at hand then a customized DOE tables can also be leveraged. Further discussion on how to utilize DOE table in real problems is addressed in Chapter 5 with the optimal maintenance problem of

a multi-unit system under dependencies. It is also examined in Chapter 5 that the maintenance problem for a single component does not require sophisticated knowledge about the DOE tables, since DOE table for this case simply means the discrete sample points for the MC simulation. The third input for MC simulation is the dependencies of lifetime distribution which can be established by examining the functional relations among components. If there are two processors in the computation unit, it is intuitive that failure of one processor will induce the increase in the failure rate of another one as in the shared load system. Last input of minimal path or cut set can be observed from RBD or transition diagram of the system, since we already have information on the physical arrangement of system. MC simulation is performed on the different combination of decision variables in DOE table with the customized VRT to increase the efficiency of simulation.

Next step is called as 'concurrent simulation'. The word 'concurrent' is used to emphasis the integration of real time simulation. Whenever the state of each component alternates between lifetime (up time) and repair time (down time), the simulated result is updated. Based on the minimal cut set or path set, the component status at each transition time would change the system status. As mentioned in the Section 3.4, the suspended animation assumption can be relaxed, if customers want the conservative result. The 'concurrent simulation' step allows the operators to choose from various shut-off rules for their applications. Underestimating availability by fully relaxing suspended animation can give safety margin for the real application, if it is required by the customers. The nature of modeling, whether it is component based or state based, usually neglects uncontrollable settings and combines minor sub-components into one component.

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Therefore, underestimated simulated result from relaxing suspended animation may not be too conservative for real applications as customers expected.

'Stopping criteria' step comes next for the simulation module. The purpose of simulation module is to generate the data sets which are not analytically tractable by mathematical approach. It is assumed that the optimal solution, i.e. optimal combination of decision variables, lies within the range of variables in DOE table. If one simulates reliability measures from 0 to a certain upper bound for the periodic PM interval, then the optimal periodic PM interval from the optimization should be within the data range. It is assumed that interpolation is accepted from discrete simulation results, and this is what next module, i.e. RSE module, is all about. For example, the simulation range for the periodic PM interval is [0,30] as in the Figure 5 for the expected number of failure during the interval. In this setting, the resulting numerical optimization has boundary condition of [0,30], and expecting the optimal periodic PM interval to fall within the range. If the optimal solution is 30 then one can not tell whether it reaches global minimum or not, since boundary condition is arbitrary selected from the simulation. The quick way to check the solution is to rerun the simulation up to a certain number greater than 30 to construct another RSE for the given range. If the solution is still 30 then it validates we had the right solution from the previous setting. Therefore, it is important to set the range as large as possible at the first hand to have the optimal solution always lies within the initial range. The Proper initial setup for the simulation bound is discussed with quasi-renewal and (p,q) maintenance model under periodic maintenance policy.

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$$\min : L(T) = \frac{c_p + c_f p^2 \sum_{i=1}^{\infty} q^{i-1} M(iT)}{T} \tag{20}$$

c_p : PM cost, c_f : CM cost

$M(T)$: renewal function

Equation (20) represents single component maintenance without considering repair time. In this section the determination of the simulation range is discussed. Other results and comments on the model will be addressed in section 5.4. By looking at the optimization problem, it is not trivial to get the lower and upper bound of the DOE table to ensure the optimal solution always lies within the range. If the range of periodic PM interval is set from 0 to very large number then it would increase the computational time from increase in inner sample points for MC simulation. That is why the proper upper bound is necessary to reduce the simulation time and to make regression process easier. The optimal solution for equation (20) changes by the relationship between c_p and c_f. If c_p is extremely smaller than c_f then it is optimal to do the PM every seconds to make the system perfect. Other extreme case occurs if c_p is considerably larger than c_f to result in the optimal PM interval to be infinity. In practical setting, PM cost is smaller than CM cost, so optimal PM interval does not go up to infinity. The rule of thumb to incorporate practical setting is to set the simulation range for PM interval from 0 to four times of expected lifetime of component when $\alpha = 1$. α is set as 1 to widen the simulation interval. It will be observed in section 5.4 that the rule of thumb for the boundary selection in MC simulation guarantees the optimal solution to be lie within the range for various maintenance cost settings.

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4.4   RSE module

The simulation results of reliability measures (e.g., expected number of failure, availability, etc.) are set as inputs for the RSE module as shown in Figure 10.
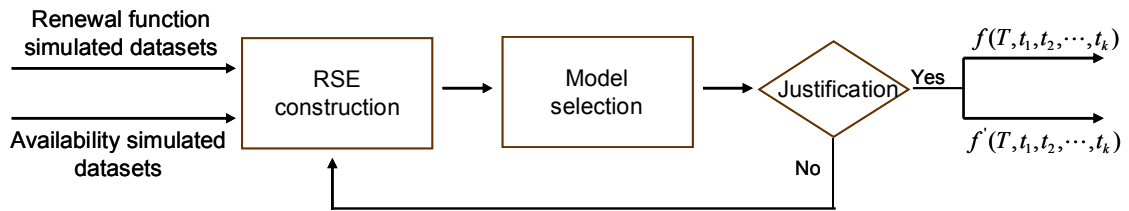
Figure 10: Flow Diagram of Curve fit / RSE Module

The simulation results from MC simulation are discrete, and it is required to have continuous functions with respect to decision variables. As mentioned in the section 3.2, if there is only one decision variable then general curve fitting is implemented to generate the representative formula from simulation results. If multiple decision variables determine the outputs of the MC simulation then multi-variate regression is used to construct RSE. The general step to construct RSE begins with polynomial equations with interaction terms. Sometimes, transformation or higher orders of polynomial are included in RSE to decrease the Sum Square of Error (SSE). Exponential or logarithm is used for the transformation of decision variables.

'Model selection' step selects the proper model from certain selection criteria. There may be several representative formulas from the 'RSE Construction' step. As mentioned in section 3.2 that the expected number of failure with respect to periodic PM interval can be represented by polynomial, exponential or other distributions which have a capable of representing increasing curvature. The goodness of fit measure, such as adjusted R- square, scattered error and so on, can be applied to select the best model. If

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likelihood is obtained from MLE then likelihood ratio can be the measure to select the model to represent the discrete simulation results.

There is one more step before finalizing the surrogate model for a given reliability measure. 'Justification' step checks monotonicity and complexity of RSEs. Monotonicity is obvious to understand from the fact that expected number of failure will increase as periodic PM interval increases, due to IFR property. Most reliability measures would have monotonicity with respect of decision variables from properties of IFR distribution of component. Complexity is also considered in 'Justification' step to have as parsimonious model as possible. The use of RSE over mathematically derived equation is to increase the reusability, since analytical closed-form equations of reliability measures tend to be complicated by integral equations. That is the reason why simple quadratic equations are first fitted with discrete simulation results. If simple quadratic equations are acceptable then these simpler equations can be easily used in further analysis, such as sensitivity analysis. Moreover, it is better to have less complicated formulas for the numerical optimization to reduce the computational burden and increase chance of reaching the optimal solution with current available optimization techniques. It should be note that the objective function, long run average cost rate, is nonlinear equation by the mathematical definition, and nonlinear optimization is regarded as one of the hardest optimization problems, if Hessian of the objective function is not semi-positive [71, 96]. Furthermore, if the decision variables should be integer values, as in failure limit policy, then the problem becomes nonlinear integer problem. Such problem is well known for exponential computational time and hard to reach the optimal solution when there are constraints. Therefore, it is better to have simpler formula as possible as from the RSE

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module. Here is the example which makes 'justification' step very important.

After the 'model selection' step the RSE with logarithm and exponential of interaction terms gives the best goodness of fit measure than the polynomial combination case. For this example, it is assumed that the complex RSE including transformations of decision variables results in an adjusted R-square 0.97, and simple polynomial RSE yields 0.95. The rest of measure is assumed to be similar between two models. Adjusted R-square is used to compensate for more parameters in the equation. The 'Model selection' step would select complex RSE over simple RSE from the goodness of fit results. It is the 'Justification' step to make decision on whether to use the complex RSE or not. The adjusted R-squares of 0.97 and 0.95 do not make big difference in accuracy of the RSE formula over discrete simulation results, but it would result in a tremendous difference in optimization module. Complex RSE tend to make typical line search algorithms to fail and force to implement heuristic approaches to solve the problem. The typical heuristic numerical optimization techniques do not have mathematical proof to reach the global optimum, and these only guarantee to reach the optimal solution if the simulation points or run time is large enough. Genetic Algorithm (GA) or Simulated Annealing (SA) methods are the common heuristic methods to solve the optimization problems with a lot of local minima or strong nonlinearity [42, 54]. It is intuitive that complex RSE will take considerably long computation time to reach the optimal solution from highly nonlinear equations involved in the numerical optimization. On the other hand, optimization problem set up by simple RSE would have more chance of being solved by typical line search algorithm such as Sequential Quadratic Programming (SQP), and optimal solution is obtained with lesser computation power.

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In conclusion, 'Justification' step will select simple RSE over complex RSE, if goodness of fit measure is close to each other. It should be remembered that the whole process of constructing surrogate models from MC simulation is to shorten the computational effort to set up the optimal maintenance policies for a complex system. Moreover, early design phases, such as conceptual and preliminary phase, tend to have the models whose geometry changes frequently from the design iteration as illustrated in Figure 1. Therefore, rapid processing is necessary to yield the optimal maintenance policies and figure of merit, such as maintenance cost rate or availability of the system, that lead better understanding of the process development loop shown in Figure 1.

4.5   *Optimization module*

The optimization problem is reconstructed by the surrogate models from the RSE module. It is proven that there is finite optimal solution that minimizes the long-run average cost rate, if the distribution of lifetime is IFR. This can be understood from equation (20) that as periodic PM interval increases, renewal function increases much faster than the renewal cycle to force the optimal decision to move away from going infinity for typical setting of PM and CM cost. The benchmark distribution is exponential that the renewal function is linear in PM interval. The long run average cost rate is constant for any PM intervals from equation (20). The reason for this is from the memoryless property of the exponential distribution. Another reason is that exponential distribution has constant failure rate which can be categorized as either IFR or DFR distribution. If DFR distribution, such as Pareto distribution, is used in this analysis then it will always give the PM interval infinity for a single component optimal maintenance problem, since there are lesser and lesser failures under DFR distribution as time passes. In summary, as long as IFR distribution is used it is guaranteed to have the optimal solution with the practical setting of maintenance cost.

Generally, numerical optimization using line search is performed based on Hessian and Gradient as in SQP programming, since global convexity of optimal maintenance problem is guaranteed by IFR distribution. If SQP cannot solve the problem then new methodology should be developed in this section.

CHAPTER 5   PRELIMINARY ANALYSES

This chapter examines validation of the proposed methodology by the numerical examples. The proposed methodology may seem reasonable from the description given by the previous chapter, but feasibility and acceptability should be tested before implementing the methodology to the real applications. If the methodology is only verified in mathematical ways then it is not different from the past and current mathematical models out there.

Through out this chapter, analyses are preformed to show the consistency with the results from the current available approaches. Moreover, the more realistic modeling setup by the proposed methodology is discussed to show how it can improve the current available approaches. Each analysis would be a building block for the final application in Chapter 6, since each analysis only focuses on the certain area. For example, Analysis 1 validates the strength of simulation of the proposed method, and Analysis 2 gives the flexibility of methodology over current maintenance scheduling in aircraft engine companies from mathematical modeling.

5.1   Analysis 1: Availability Simulation

In this section, availability is simulated from SPN with aging token developed by Georgia Tech Research Corporation (GTRC) and Dr. Volovoi to compare the results from the proposed methodology. The availability is an important measure that is generally used as one of constraints in the optimization maintenance problem.

A Petri net provides a graphical representation of system's states based on places

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and tokens. Tokens represent the states at the current time, and token moves to other place by transition rules [76]. A Petri net is very similar to Continuous Time Markov Chain (CTMC), but it is more generalized by the firing rules in transitions. The original idea of Petri net was introduced by Dr. Petri in 1962 with his PhD thesis. The original Petri-net was deterministic that the firing rules were given at the beginning of simulation. There were a lot of improvements over the original model since then to capture more practical applications, such as dependencies in failure and repair, different transitions by different tokens (colored Petri net) and aging in transitions [44, 60, 97]. The system in Figure 11 is analyzed to compare the availability simulation results from methodologies.
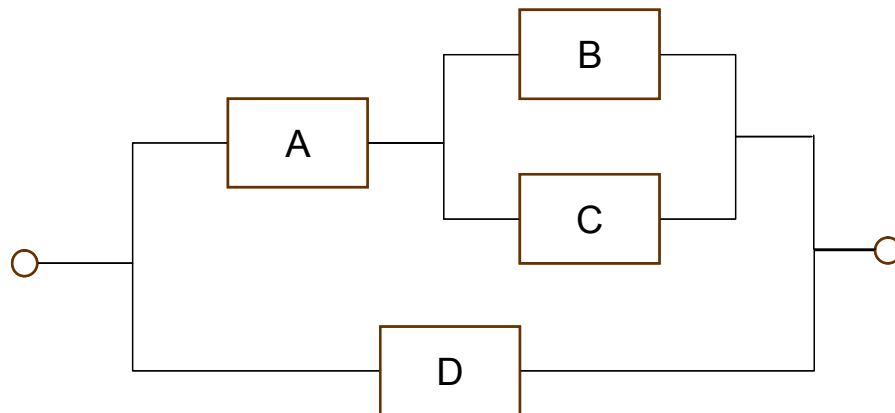
Figure 11: RBD of System for Availability Simulation

Table 4: Distribution for Each component for Availability Simulation

Component	Distribution
A	Weibull distribution (10,1.5)
B	Uniform distribution (15,20)
C	Weibull distribution (10,2)
D	Exponential distribution (8)
Repair	Exponential distribution (2)

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The main goal of Analysis 1 is to compare the result of SPN with aging token simulation with the result from the proposed methodology. SPN with aging token (hereafter called as SPN@) has many features to enable automatic simulation. The software has libraries of different distributions and capability of simulation under two different modes, i.e. steady state and transient. In this analysis, to make a comparison under the same setting for both methodologies, transient simulation time is set as 1000. Figure 12 illustrates the set up of the SPN@ to calculate the availability of the system shown in Figure 11. The diagram can be simpler, if more colored tokens are used to indicate the locality of system state.
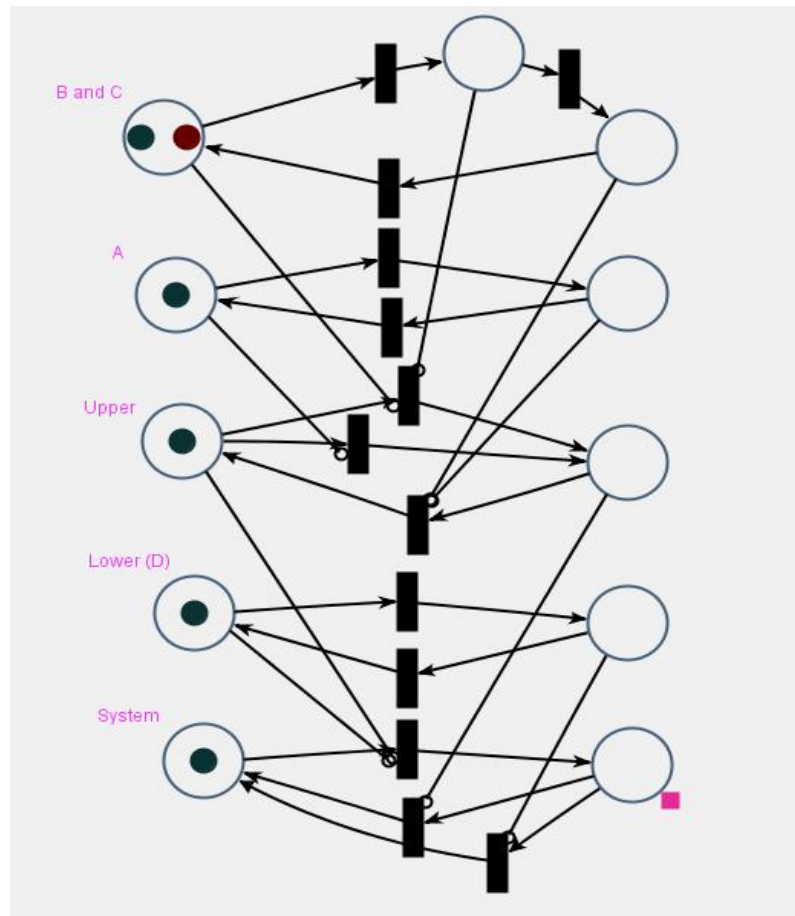
Figure 12: SPN@ Representation of Analysis 1

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It can be observed that SPN@ usually does not require $2^4 = 16$ states to represent the system as in CTMC. One extra circle for component B is from the SPN@ set up which only can simulate uniform distribution [0,t]. There is a state related to the sensor to count the system failure. The minimal cut set are $\{A,D\}, \{B,C,D\}$, so whenever combination of components' state is in the minimal cut set, system is in failure state. Also Figure 12 illustrates how the structure property of RBD, such as parallel or series system, can be represented by SPN@. Repair time from exponential distribution is spent for the failed components. It does not need to have the repair time for each component to be identical as in this analysis. The goal of this analysis is to compare the availability of the system, so varying repair time would result in different system availability. Table 5 shows the result from SPN@ and the proposed methodology.

Table 5: Result for Analysis 1

	SPN@	The proposed methodology
Average availability	$1 - 0.03928 = 0.9607$	0.9599
CPU time	7~8 Sec	2.5 Sec

SPN@ simulates average unavailability from probability of token at system failure. That is why average availability is calculated from subtracting unavailability form 1. The inhibitor and enabler provide the logics of transitions much simpler than the approach from the original Petri net.

The simulated results for average availability from both methodologies give very close answer. The difference in CPU time is not very important in this analysis. The difference is anticipated from the Graphical User Interface (GUI) of SPN@ which may require some computational power during the simulation. Simulation from the proposed

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methodology is written in MATLAB not to have any GUI features to increase the computational efficiency.

Another difference occurs from the variation of the simulation. Both methodologies implement Monte Carlo simulation, but the proposed methodology is capable of incorporating customized VRT as mention in section 2.2.2. The naïve way of setting Confidence Interval (CI) for the simulation can be applied here for the comparison. Naïve CI method is performed by ordering the simulated result form the lowest to the highest. Then, 90% CI would be the range of simulated result form the fifth smallest one to 95th largest one, if 100 samples are obtained. It is observed that SPN@ results tend to have larger CI than the one from the proposed methodology when the total number of sample is fixed. 100-sample CI comparison requires time consuming manual work, so only 30 samples are used to construct naïve CI.

Besides the differences in the results, there is an important improvement in the proposed methodology. SPN@ can model imperfect maintenance by introducing ages in the token, but successive aging can be represented by more tokens and new states. The resulting diagram would huge, and the graphical interface may have disadvantage over debugging the whole system. On the other hand, the proposed methodology implements quasi-renewal processes for the imperfect maintenance as discussed in Chapter 4. Whenever maintenance is performed, the scale parameter in lifetime distribution is automatically multiplied by the factor to represent imperfect maintenance. The imperfect maintenance keeps on going until predetermined simulation time is reached.

Figure 13 is generated by simply changing the α factor in the proposed methodology to capture the influence of imperfect maintenance over availability.

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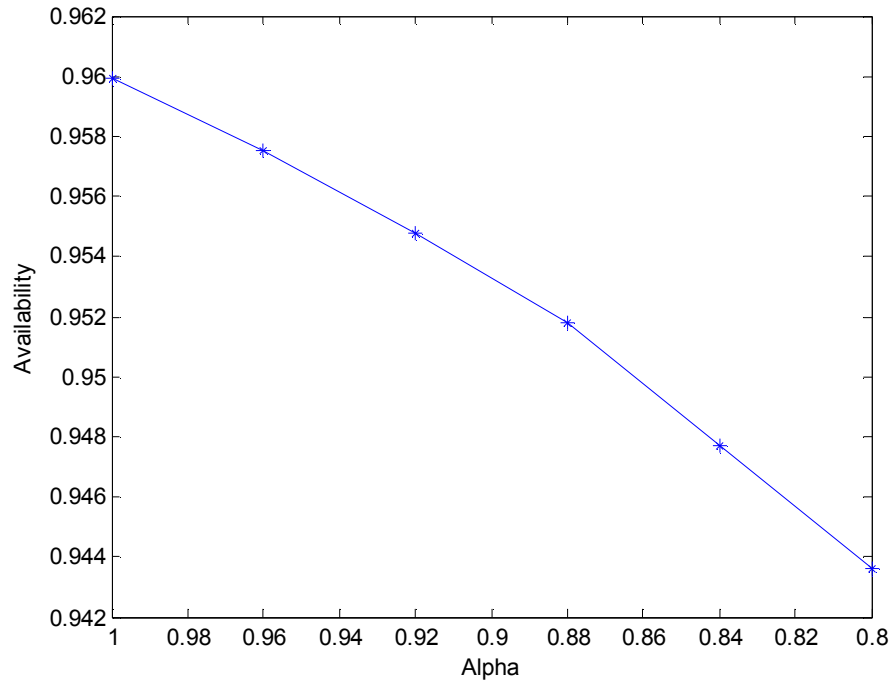
Figure 13: Successive Imperfect Maintenance and Availability of Analysis 1

The perfect repair is the case when α is 1. If the quasi-renewal factor, α, starts to decrease then the expected lifetime of component becomes shorter and shorter. If simulation time is considerably larger than expected lifetime of system or components then there would be more chances of repairing the system or components. This will result in the increase in sensitivity of average availability with respect to quasi-renewal factor, since there is tendency for the failure to occur more frequently from IFR or Increasing Failure Rate in Average (IFRA) distribution. Similar argument can be established with β factor to represent a longer repair time after a successive imperfect maintenance.

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5.2   Analysis 2: Inspection Policy for a Single Component

In this section, currently performed inspection policy is compared with the proposed methodology. Typically, inspection policies mentioned in the literature are little different from maintenance policies under the optimal maintenance problem, but the basic idea is similar. Brief description of inspection model is discussed here.

5.2.1   Current Maintenance Policy for Long Term (LT) Failure of FADEC

Full Authority Digital Engine Control (FADEC) system is very important system to control the engine thrust based on the input signal. The failure of FADEC system results in Loss of Thrust Control (LOTC), and the vehicle should make an immediate landing to avoid further catastrophic failure. The failure of FADEC is divided by Short Time (ST) failure and Long Time (LT) failure depending on the chance of failure. The convenience of dividing the failure into two categories is to help understanding system better by the concept called as Time Limited Dispatch (TLD) [77]. If component is among ST dispatch group then the ST dispatch interval is used for the repair. The general idea can be modeled under CTMC in Figure 14 [32]. More detail description of TLD of FADEC is stated in Chapter 6.
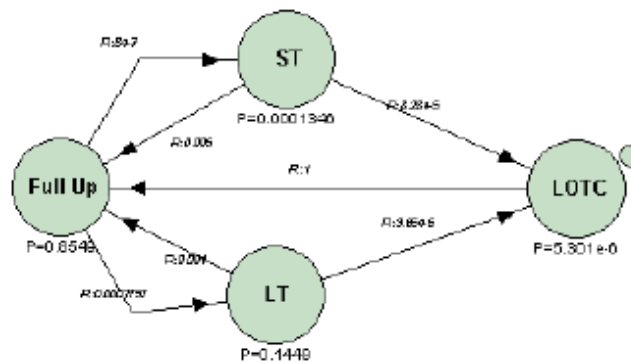
Figure 14: Markov Chain Representation of TLD of FADEC

The CTMC assumes the repair at the failure. This is called on-condition repair which turns out to be suitable for Markov Chain, since there is no delay in transition from one state to another state in the model [31]. FADEC system allows ST component to be repaired on failure due to the indicator, and this maintenance policy is called as Minimum Equipment List (MEL) policy. On the other hand, LT failure usually does not have indicator to detect the failure, and periodic inspection is performed to check for the LT component state. Practitioners in industries perform the general analysis by CTMC to get the LOTC failure rate and find LT dispatch interval to meet the required LOTC failure rate for given ST dispatch interval [32]. As one can expect, it is required to relate LT dispatch interval with LT inspection interval to set up the periodic inspection for the LT components. Practitioners assume there is gap between time of failure and inspection interval and denote this interval as time since failure. The idea of inspection and LT dispatch is shown in Figure 15.
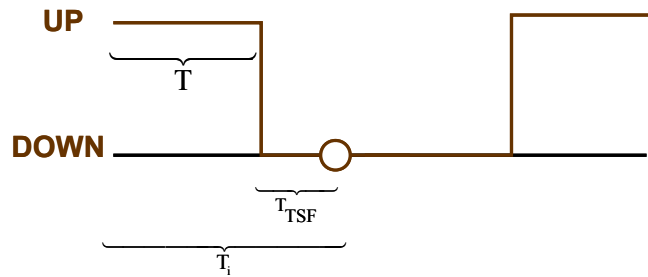
Figure 15: Diagram for Inspection, LT failure and dispatch

The mathematical representation of time since failure (T_{TSF}) can be written as follows with lifetime (T) and inspection interval (T_i).

$$T_{TSF} = \min(T_i - T, 0) \tag{21}$$

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The industrial approaches try to set the expected value of time since failure ($E[T_{TSF}]$) as predetermined value and find the inspection interval. Generally, the predetermined value is based on LT dispatch interval or any interval from maintenance experience. Obtaining expected value of time since failure by past empirical data is usually determined by the upper bound given by the regulations [33]. The reasoning from experiences generally results in biased decision, and it would be discussed in this chapter why we should avoid such approaches. The expected value of time since failure for general distribution, if the inspection has occurred, can be calculated as in equation (23).

$$E\left[T_{TSF}\right] = T_i - E(T \mid T \le T_i)$$
$$= T_i - \frac{\int_0^{T_i} \tau f(\tau) d\tau}{1 - S(T_i)} \tag{22}$$

If lifetime is exponential with rate λ then equation (22) becomes as follows.

$$E\left[T_{TSF}\right] = T_i - \frac{\left(\dfrac{1}{\lambda} - \left(\dfrac{1}{\lambda} + T_i\right) e^{-\lambda T_i}\right)}{\left(1 - e^{-\lambda T_i}\right)} \tag{23}$$

Analysis 2 assumes the expected value of time since failure to be 50 by engineering judgment, and inspection interval can be obtained by numerically solving equation (23) [24]. This method relies too much on the engineering judgment and experience, since time since failure is just assumed to be 50 by assumption. Besides the subjective design making from equation (23), there are problems with the approach from distribution and inspection cost point of view. If Weibull or other types of distribution is used, the integral in the left hand side of equation (22) is not simple to solve. Moreover, it

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is intuitive that the inspection interval should be the function of inspection cost. Next section discusses a modified approach to the problem.

5.2.2 Modified Approach for FADEC LT Failure Maintenance

Section 5.1 discussed the strength of simulation capability, and this section addresses the mathematical capability of the proposed methodology. It would be studied that mathematical modeling can eliminate subjective decision making and allow operators to have more degree of freedom for the maintenance policies based on various cost settings. Chapter 6 readdresses finding optimal maintenance policies for TLD of FADEC system under the optimal maintenance problem which sets up numerical optimization problem to satisfy the constraint while having a minimum cost rate.

Modified approach utilizes the operational expense distribution instead of relying on the engineering judgment to set the time since failure. Expense distribution is the function of operational expense with respect to time since failure. Right after the LT component failure, it does not cost much to the operator, but as time passes without repairing the component, the expense cost accumulates. It is assumed that the accumulation of cost increase exponentially after certain period time to force the operator to perform CM on the component. The accumulating cost with respect to time since failure can be expressed in Figure 16.

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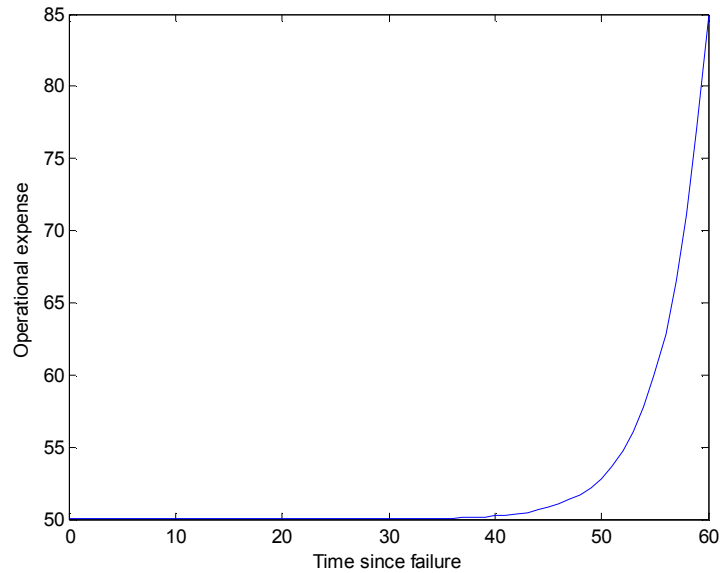
Figure 16: Operational Expense Distribution with Time Since Failure

The expense distribution is assumed to have sharp increase at 50 to be consistent with engineering judgment of $E[T_{TSF}] = 50$. From this setting, it is assumed that the operator's subjective decision is not very biased. Nevertheless, different results from the proposed methodology are provided to suggest that the subjective decision making is not flexible to capture changes in operational environment.

The expense curve can be generated from the data given by statisticians. The statisticians can gather the historical data to generate the curve as shown in Figure 16. If there are not much data to generate the distribution, statisticians can perform experimental simulation, such as Accelerated Life Testing (ALT), to shorten the lifetime and gather more information about the operational expenses. Further discussions on experimental simulation lie outside the scope of this thesis, but there are many literatures on performing ALT or inference on ALT [59]. The operational cost in Figure 16 has following equation.

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operational cost: $50 + 10\exp\left(\dfrac{t - 1.1T_{TSF}}{4}\right)$ (24)

Based on the operational cost function, total cost function can be constructed as in equation (25).

$$C = \begin{cases} c_i N(t), \ t \geq T_i \\ c_i + f(T_i), \ t < T_i \end{cases}$$

c_i : inspection cost (25)

$N(t)$: # of inspection during [0,t]

$f(T_i)$: operational expense function

Equation (25) accounts for two cases. First case indicates the inspection happens before LT failure, so the operator is going to spend money on inspection only. Second case is that the inspection happens after failure, and because of the difference between inspection interval and LT lifetime, i.e. time since failure, there would be the expense from failed LT component. The expense occurred during time since failure is modeled by the equation (24). The expected total cost function is shown in equation (26), and this will be used for determining the optimal inspection interval for LT component.

$$\begin{aligned} E[C] &= E\big[C|t \geq T_i\big]P\big(t \geq T_i\big) + E\big[C|t < T_i\big]P\big(t < T_i\big) \\ &= c_i \frac{E\big[t|t \geq T_i\big]}{T_i}P\big(t \geq T_i\big) + E\big[f(t)|t < T_i + c_i\big]P\big(t < T_i\big) \end{aligned}$$ (26)

Optimizing equation (27) will give the optimal inspection interval.

$$\begin{aligned} &\min : E[C] \\ &s.t. \ T_i \geq 0 \end{aligned}$$ (27)

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The given approach is one type of optimal inspection policy that minimizes total expected cost until detection of failure [5]. One can use cost rate for this inspection problem, but it will give constant for cost rate for any intervals because of exponential distribution.

The result from both methodologies is tabulated in Table 6 with exponential distribution of $\lambda = 0.001$ and inspection cost (c_i) of $100.

Table 6: Result for Analysis 2

	Industrial approach	Proposed methodology
Approach type	Deterministic	Probabilistic
Inspection interval	333.81 days	351 days
Total cost	$328.17	$318.28

The total cost for traditional methodology is calculated by setting inspection interval as 333.81 in the optimization equation.

5.2.3 Comment on the Result

The proposed methodology does not require engineering judgment on the expected time since failure, and rely on the actual distribution of operational expense from historical data. The time since failure from the proposed methodology is 55 not 50 which means we can wait another 5 days before performing the inspection. If the distribution parameter for expense curve is changed by more reliable manufacturing or materials, one should not rely on the engineering judgment on the expected time since failure. The mathematical modeling from the proposed methodology gives expected time since failure as an output from the optimization, not an input from engineering judgment. The only input for the proposed methodology is the expense curve, which can be

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modeled from real data by statisticians.

Another advantage from the proposed methodology is the mathematical set up which considers the inspection cost. The difference of approximately $10 is not big enough, but it should be note that this difference heavily depends on the inspection cost per failure. As inspection cost increases, it will shift inspection interval, and the difference between deterministic and probabilistic approach may vary a lot. Of course, the $10 difference is per failure. The cumulative cost for a given number of failures during the life time of a system would not be small enough to be ignored.

In conclusion, Analysis 2 shows how proposed methodology on the maintenance can be solve the inspection problem. The mathematical capability of the proposed methodology to interpret the inspection problem ensures more a reliable solution than the maintenance police beginning by the subjective decision making on input parameters as in industrial approaches. The maintenance scheduling for TLD of FADEC under optimal maintenance problem would be further discussed in Chapter 6, where LT inspection interval is optimized by the responses from the whole system not just from the component level as in this analysis.

5.3   *Analysis 3: Replacement Policy for a Single Component*

This section discusses how traditional way of replacement may not be optimal in the general setting. The alternative replacement policies are the age dependent policy and Block replacement policy. The formulas and properties of two replacement policies are given in section 2.1.3.

The general methodology for replacement policy in practice is the failure induced replacement. Practitioners sometime have little knowledge on the lifetime distribution or are not informed about various replacement policies. Unlike repairable components, there is no direct regulation on the nonrepairable components. Some of the nonrepairable electrical components, such as fuss or filament in the light bulb, tend to follow exponential distribution. If the nonrepairable component follows exponential distribution, the cost rate from various replacement models would be same, due to memoryless property. Under more general settings, one may have following questions. What if the lifetime distribution is not exponentially distributed? What if we should prevent the upcoming failure, since it induces the catastrophic failure of the system?

These questions lead us to revisit failure replacement policy with other replacement policies. Typically, comparing among policies is not recommended because of different settings in the imperfect maintenance models. For the case of perfect maintenance policy, i.e. replacement policy, different polices can be to be analyzed.

The goal of this section is to show the mathematical capability of the proposed methodology over the traditionally performed replacement policy. Moreover, it is discussed that mathematical set up in the proposed methodology allows operator to have many options for their replacement policy models.

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5.3.1 Current Replacement Policy for a single Component

The failure replacement policy is generally performed in practice, because of number of spare units is limited or because of insufficient information about various replacement policies. The occurrence or replacement from various policies is shown in Figure 17 by Leemis (1995) [59].
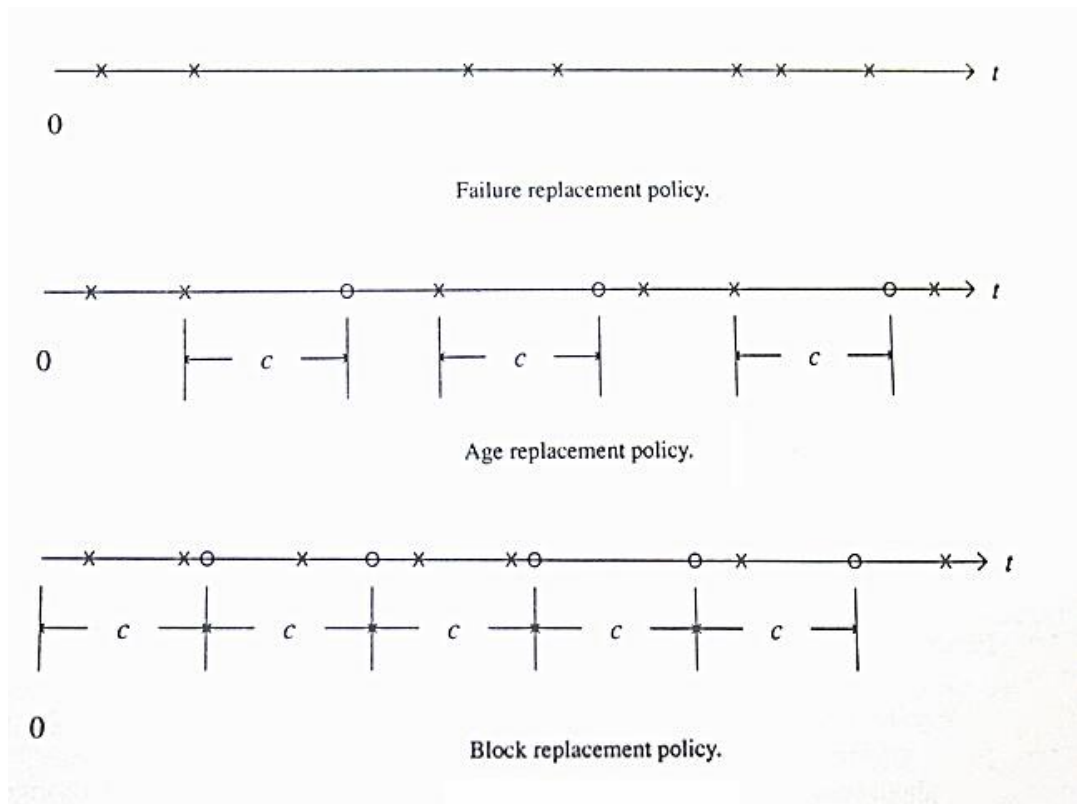
Figure 17: Replacement Occurrence from Various Replacement Policies

The expected number of items consumed by time t under various models can be expressed as follows [59], if the spacing c in Figure 17 is constant.

$$E[n_f(t)] \leq E[n_a(t)] \leq E[n_b(t)], \ t > 0 \tag{28}$$

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From equation (28), it is true that expected number of consumed nonrepairable item by failure replacement policy is smaller than the ones from other replacement policies. Nevertheless, expected number of item consumed can not be the measure of selecting the policies, because it does not give any information about the long-run average cost. As mentioned in previous sections, it is more reasonable to consider the long-run effect of maintenance cost to capture the life cycle cost of the item.

The comparison among replacement policies is performed based on the long-run average replacement cost as defined in equation (12). Also Gamma distribution of rate $\lambda = 0.01$ and shape parameter $\kappa = 2$ is assumed for the lifetime of a nonrepairable component for Analysis 3. The reason for choosing Gamma $(0.01, 2)$ is from the analytical tractability of expected number of failure by time t.

Let c_1 denote the replace cost when item is failed, and c_2 be the cost when item is working as defined in section 2.1.3. Then, the long run average cost rate for the failure replacement policy would be only related to c_1, since replacement only happens at the component failure.

$$L(t) = \frac{c_1}{E[T]} = \frac{c_1}{(2/0.01)} = \frac{c_1}{200} \qquad (29)$$

The lifetime is denoted as random variable T and expected value of Gamma distribution is $\frac{\kappa}{\lambda}$ as shown in equation (29). More general property of Gamma distribution is discussed by Hayter (2002) [39].

5.3.2 Age Dependent Replacement Policy for a Single Component

Equation (13) and (14), in section 2.1.3.1, give the optimization problem and optimality condition under age dependent replacement policy. It is true that Gamma function is less popular than Weibull distribution partially due to the intractability of the survivor function. Gamma function in this analysis becomes Erlang distribution, because of integer value of shape parameter. Erlang distribution (λ, n) is the n convolution of identical and independent (iid) exponential distribution. The survival function and hazard function of Erlang distribution (λ, n) is shown below to obtain the optimality condition.

$$S(t) = \sum_{k=0}^{n-1} \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

$$\text{if } \lambda = 0.01, \ n = 2: \ S(t) = e^{-0.01t} + 0.01t e^{-0.01t}, \ r(t) = \frac{0.01^2 t}{1 + 0.01t}$$

(30)

Then optimality condition and the resulting cost rate are shown as follows.

$$\frac{0.01^2 t^*}{1 + 0.01t^*} \left[\frac{2}{0.01} - \frac{2}{0.01} e^{-0.01t^*} - t^* e^{-0.01t^*} \right] - 1 + e^{-0.01t^*} + \lambda t^* e^{-0.01t^*} = \frac{c_2}{c_1 - c_2}$$

$$L(t^*) = \frac{c_1 F(t^*) + c_2 S(t^*)}{\int_0^{t^*} S(t)dt}$$

(31)

The optimality condition can be numerically solved, and optimal cost rate can be obtained. The result is shown in section 5.3.4.

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5.3.3   Block Replacement Policy for a Single Component

Unlike age dependent replacement policy, Block replacement is based on the predetermined periodic interval. Equation (15) and (16), in section 2.1.3.2, give the optimization problem and optimal criteria. To solve the optimization problem, it is required to have prior knowledge about renewal function, $M(t)$. It is mentioned that renewal function for IFR distribution is not trivial to obtain, and the proposed methodology obtain this value from constructing RSE from simulated data. Analysis 3 is controlled to have a distribution which has renewal function in closed-form. Solving for the renewal function by Laplace-Stieltjes transform is shown in equation (32) [110].

$$\text{Define: } m^*(s) = \int_0^\infty e^{-st} dm(t) \text{ and } F^*(s) = \int_0^\infty e^{-st} dF(t)$$

$$
\begin{aligned}
M(t) &= F(t) + \int_0^\infty M(t-u)dF(u) \\
&= F(t) + M * F(t) \\
m^*(s) &= F^*(s) + m^*(s)F^*(s) \\
m^*(s) &= \frac{F^*(s)}{1 - F^*(s)}
\end{aligned}
\tag{32}
$$

Based on the distribution parameters, i.e. $\lambda = 0.01, \kappa = 2$, the optimal condition and optimal cost rate is shown in equation (33).

$$
M(t) = 0.25e^{-0.02t} + 0.005t - 0.25, \ M'(t) = \frac{d}{dt}M(t) = -0.005e^{-0.02t} + 0.005
$$

$$
t^* M'(t^*) - M(t^*) = \frac{c_2}{c_1}, \ L(t^*) = \frac{c_1 M(t^*) + c_2}{t^*}
\tag{33}
$$

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5.3.4 Comment on the Result

The replacement cost if the component is failed (c_1) is set as 1000 and the cost when the component is operating (c_2) is set as 100. This is the usual case that c_2 is smaller than c_1, due to salvation or resale price of operating component. The cost rate and optimal interval of a single component with Gamma (0.01, 2) is shown in Table 7.

Table 7: Result for Analysis 3

Replacement policy	Failure	Age dependent	Block
Optimal interval	200	68.01	68.82
Cost rate	5	3.6433	3.7376

As expected, age dependent and Block replacement policy result in lower cost rate than the failure replacement policy. It should be noted that if optimal interval under age dependent and Block replacement are infinity then the policies become failure replacement policy. The existence of finite optimal solution may vary by the combination of c_1 and c_2. The assumption in Analysis 3 is that c_1 is ten times expensive than c_2. If $\frac{c_1}{c_2} \leq 4$ then it is optimal for Block replacement policy becomes failure replacement policy for Gamma with shaper parameter 2. Other distribution would have different the cost ratio, $\frac{c_1}{c_2}$, to have finite solution for Block replacement policy. The conclusion is same for age dependent policy, but this policy guarantees unique and finite solution, if failure rate, $r(t)$, is strictly increasing to infinity. Weibull with shape parameter greater than 1 or truncated normal distribution can be under this category to have the unique and

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finite solution [7]. If unique and finite solution (t^*) exists then optimal cost rate in equation (31) becomes much simper form as in equation (34).

$$L(t^*) = \frac{c_1 F(t^*) + c_2 S(t^*)}{\int_0^{t^*} S(t)dt} = (c_1 - c_2)r(t^*) \tag{34}$$

The achievement from Analysis 3 is not to set the optimal conditions for each replacement model. The proposed methodology successfully shows the mathematical strength in the practical application. Practitioners tend to just replace nonrepairable item upon failure, but it is shown in this analysis that it is not always optimal to pursue the failure replacement policy. Most of mechanical and electrical items in electro-mechanical system, such as aircraft or rotorcraft, have IFR distribution that more chance of failure would be expected from wear out phase. It can be observed that the long-run average cost rate from age dependent or Block replacement is usually lesser than the one from failure replacement policy under IFR distribution with practical replacement cost settings. Moreover, it is not hard to generalize age dependent and replacement policy to include random cost and binomial decision between repair and replacement [12, 13, 89].

It should be noted that the special case of optimal maintenance policy is the replacement policy, and the proposed methodology is also adaptable to solve for multi-component replacement problem. If there is multi-component with economic dependence in replacement, it is definitely advantage to pursue the propose methodology over replacing each item upon failure. Because the proposed methodology set up the problem under mathematical modeling to be more flexible with parameters such as maintenance cost settings and different types of lifetime distributions.

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5.4 *Analysis 4: Single Component Periodic Maintenance*

Previous sections discuss about advantages from the proposed methodology. Simulation module is examined and mathematical setup of the problem is studied to compare the traditional approach with the proposed one. The whole processes from Chapter 4 are applied to the generalized periodic maintenance for a single component though Analysis 4. Following sections would expand the analysis to the multi-unit system maintenance problems.

5.4.1 Periodic Maintenance When Repair Time is Neglected

The model assumes imperfect CM under quasi-renewal process (α, β) and imperfect PM with (p, q) policy. The performed maintenance is modeled as imperfect to capture as much of the practical aspect of real applications. PM makes system to a better condition than the current operating status. For example, a mechanical part can be lubricated or tuned up to be very close to the perfect condition. It is common to set the p value in (p, q) policy close to 1 to include the replacement (i.e., perfect maintenance) happens during PM interval. General setting of p is 0.95 suggests that the component lifetime distribution is not influenced by the action of PM, as in minimal repair, by only small probability of 0.05.

The given problem can be the baseline for the research of this thesis, since it deals with the generalized single component maintenance. The generalized single component maintenance problem is graphically represented as a green box in Figure 4. If the proposed methodology is validated to solve the problem without any assumptions on the component characteristics then more components can be included to represent multi-

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component maintenance problem. The first half part examines the case when the repair time is neglected, and the rest of Analysis 4 would cover the situation if the repair time is to be considered.

If repair time is neglected Wang and Pham (1996b) showed the closed-form of optimization problem and optimality criteria as in equation (35) and (36) [104]. It should be noted that neglecting repair time during CM can be very restricted assumption.

$$
\begin{aligned}
&\min : L(T) = \frac{c_p + c_f p^2 \sum_{i=1}^{\infty} q^{i-1} M(iT)}{T} \\
&s.t. \ \ T > 0 \\
&c_p : \text{PM cost}, c_f : \text{CM cost} \\
&M(T) : \text{renewal function}
\end{aligned}
\tag{35}
$$

$$
\sum_{i=1}^{\infty} q^{i-1} \left[iT^* m(iT^*) - M(iT^*) \right] = \frac{c_p}{c_f p^2}, \ m(T) = M'(T)
\tag{36}
$$

As mentioned in the previous sections, the renewal function, i.e. the expected number of failure ($M(iT)$), may not have analytical closed-form. The Gamma distribution with order 2 or any integer, Erlang distribution, used in Analysis 3, is very special case to have closed-form for the renewal function. Other IFR distributions usually fail to have closed-form analytical solutions, and numerical approximation or alternative approximations by series sum should be performed. The imperfect maintenance parameters and distribution for lifetime and repair are given in Table 8. These input parameters are applied to both methodologies, i.e. current mathematical and the proposed, to compare the results.

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Table 8: Parameters for Analysis 4

	Parameters
Imperfect maintenance	$(\alpha, \beta) = (0.95, 1)$ and $(p, q) = (0.95, 1)$
Lifetime distribution	Weibull (10,2)
Repair time distribution	N/A

Since the renewal function for Weibull distribution (10,2) can not be directly obtained from renewal equation by Laplace-Stieltjes transform, MC simulation and regression analysis is performed as in the proposed methodology to capture the information on the renewal function, i.e. expected number of failure. As discussed in Section 4.3, DOE table for the problem with single decision variable is not hard to create. Here, equally spaced 8 sample points are selected for MC simulation. Then, curve fit is performed through the discrete simulation results as shown in Figure 18.
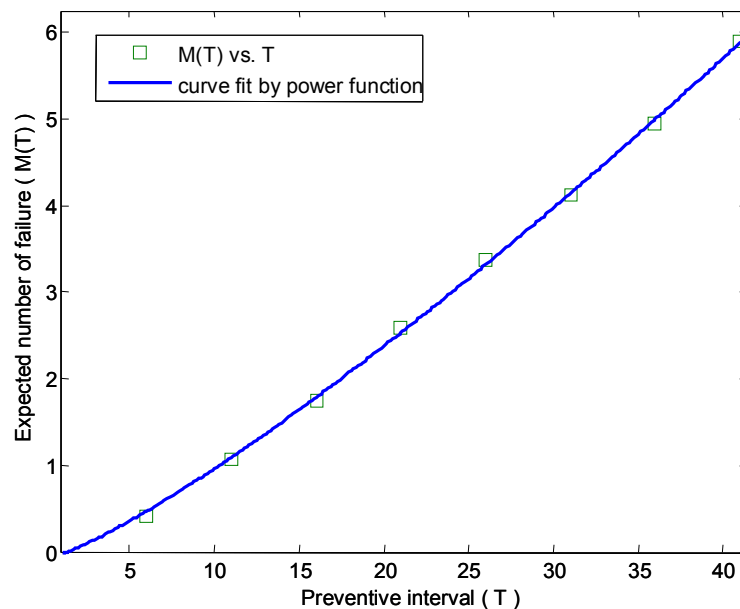

Figure 18: Renewal Function Curve Fit for Analysis 4 without Repair Time

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The resulting fitted equation is $y = 0.06188x^{1.23} - 0.08999$ by power distribution. R-square is 0.9995 and Sum of Squares Error (SSE) is 0.01788. It can be concluded that the goodness of fit measure is reasonable to accept the distribution. In this analysis, other distributions such as polynomials and bivariate exponential are also examined. The maximum likelihood and R-square for these distributions are lower than the power distribution.

The upper bound of 40 for PM interval in MC simulation is selected by the rule of thumb mentioned in section 4.3. The expected lifetime when $\alpha = 1$ can be calculated as 8.86 from equation (11). The MC simulation upper bound of 40 lies between 4 and 5 times of expected lifetime which implies there are about 4 or 5 failures during the simulation interval. If $\alpha < 1$ is used as in this analysis, there would be more failures during the interval. The rule of thumb is sufficiently satisfied with the choice of 40 time unit for range of DOE table, and it will be shown that even if maintenance cost changes the solution lies well within the interval of [0,40]. Following two sections illustrates two different cost settings to result in different maintenance policies.

5.4.1.1 CM Cost is $4500 and PM Cost is $1500

The maintenance cost setting represents relatively small PM cost. The condition satisfies existence of a finite solution for Block replacement under Gamma with order 2 from Analysis 3. Since only one decision variable, periodic PM interval, is involved in optimization problem, the objective function value, long-run average cost rate, can be plotted as the function of PM interval as in Figure 19. It should be noted that if more than 2 decision variables are involved with optimization problem, as in some of the optimal maintenance problems with multi-unit system under dependencies, then the response of

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the objective function with respect to decision variables can not be plotted.
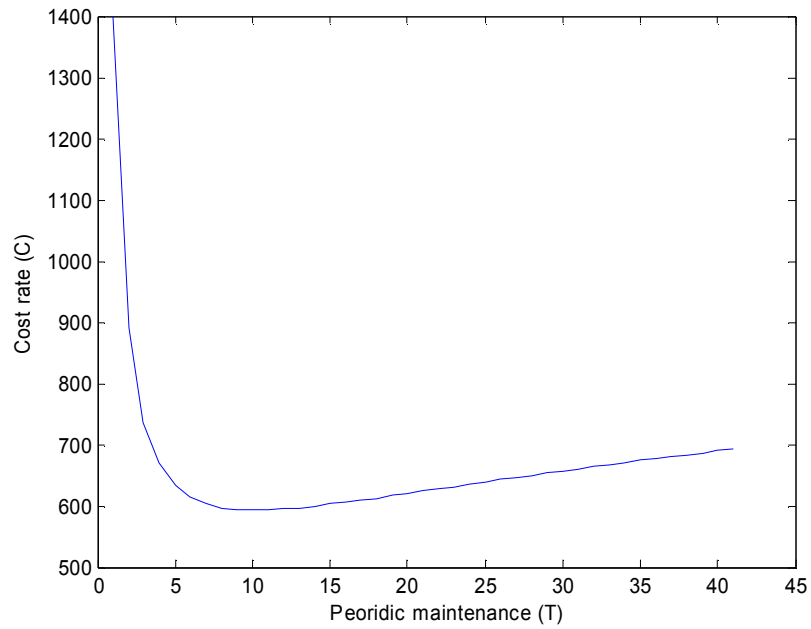
Figure 19: Simulated Result for Analysis 4 without Repair Time (Cost Setting 1)

The result of optimization by typical nonlinear optimization algorithm, SQP, is shown in Table 9.

Table 9: Result for Analysis 4 without Repair Time (Cost Setting 1)

Optimal interval	Cost rate
10.0614	592.8022

The graph in Figure 19 is to show the convexity of the objective function as periodic maintenance interval increases. The optimal PM interval is greater than the expected lifetime of the component.

5.4.1.2 CM cost is $4500 and PM cost is $3000

This cost setting indicates that PM cost is increased but still smaller than CM cost

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to capture the practical setup for most mechanical and electrical components. The simulated graph is shown in Figure 20.
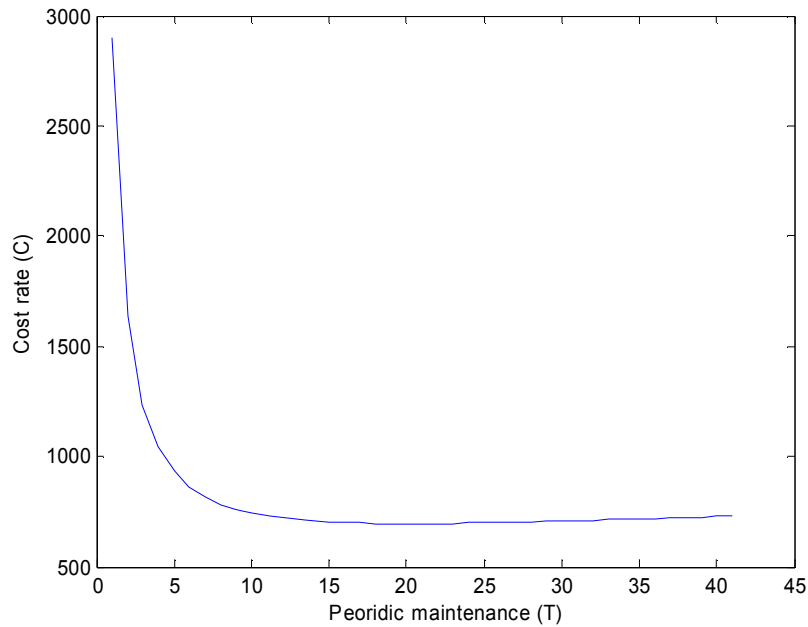
Figure 20: Simulated Result for Analysis 4 without Repair Time (Cost Setting 2)

Table 10: Result for Analysis 4 without Repair Time (Cost Setting 2)

	PM interval	Cost rate
Optimized	20.1176	695.2187
From previous setting	10.0614	742.8048

It can be observed that optimal PM interval is larger than previous setting. This is intuitive that the operator will prolong the PM because the cost of PM has been increased. If maintenance is pursued with the same interval as in the previous cost setting then there will be about $50 more cost to be paid per unit of time. If the component has been operated for 20 hours then approximately $1000 is spent because of non-optimal maintenance scheduling.

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The analysis can be extended to varying PM cost until it is reasonable to only perform CM. The optimality criteria can be examined for this type of analysis as done in Analysis 3 with replacement policy.

MC simulation provides the response of reliability measure for a given problem (i.e., expected number of failure) without imposing any assumptions on the system. Then, the simulation results are regressed by curve fit, and the surrogate model is implemented to the optimization problem to search for the optimal PM interval. Furthermore, the surrogate model of expected number of failure not only ensures rapid processing for the optimization, but also increases reusability for other analysis such as sensitivity analysis.

Next section discusses the case when the repair time is considered. Availability constraint is included in the following analysis. It is obvious that availability under negligible repair time of a single component is always 1. Even with the single component case, mathematical derivation for a closed-form formula of availability in terms of decision variable (i.e., PM interval) is not trivial.

5.4.2 Periodic Maintenance When Repair Time is Considered

If repair time is neglected then optimization problem can be written by renewal function which describes expected number of failure (CM) during the periodic interval as in previous example. If repair time is considered then the expected number of failure would be decreased for the fixed time interval. Having input parameters same as the previous example, only repair time is introduced with exponential with mean 2. It can be simple to derive that the policy without PM and quasi-renewal parameters $(\alpha, \beta) = (1,1)$ would result in long-run average availability as follows.

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$$A(t) = \lim_{t \to \infty} \frac{a(t)}{t} = \frac{E[X]}{E[X] + E[Y]}$$

$a(t)$: availability up to time t

X : lifetime r.v. Y : repair time r.v.

(37)

The equation (37) is as same as the availability defined by MTBF and Mean Down Time (MDT) in the qualitative reliability references [92]. Equation (37) indicates that renewal cycle is the sum of expected value of up time and down time. If perfect PM is performed then renewal cycle can be expressed as in equation (38).

$$A(t) = \lim_{t \to \infty} \frac{a(t)}{t} = \frac{up(D)}{D}$$

$up(D)$: expected uptime $[0, D]$

D : renewal cycle $= T +$ expected time to PM

(38)

The mathematical approaches try to have closed-form equation for the expected uptime. Tailored assumptions for mathematical derivation lead the resulting model only valid for a certain condition. The proposed methodology is suitable for the generalized cases, since it does not restrict its application by the assumptions or simplification to increase the mathematical tractability. The method directly constructs representative formula for the expected system uptime during the renewal cycle by the help of MC simulation and the RSE construction mentioned in Chapter 4.

First, expected number of failure is fitted as in Figure 21, since this measure is involved in constructing the objective function which is long-run average cost rate. The MC simulation time was manageable for this analysis, so equally spaced 40 sample points are used for DOE table.

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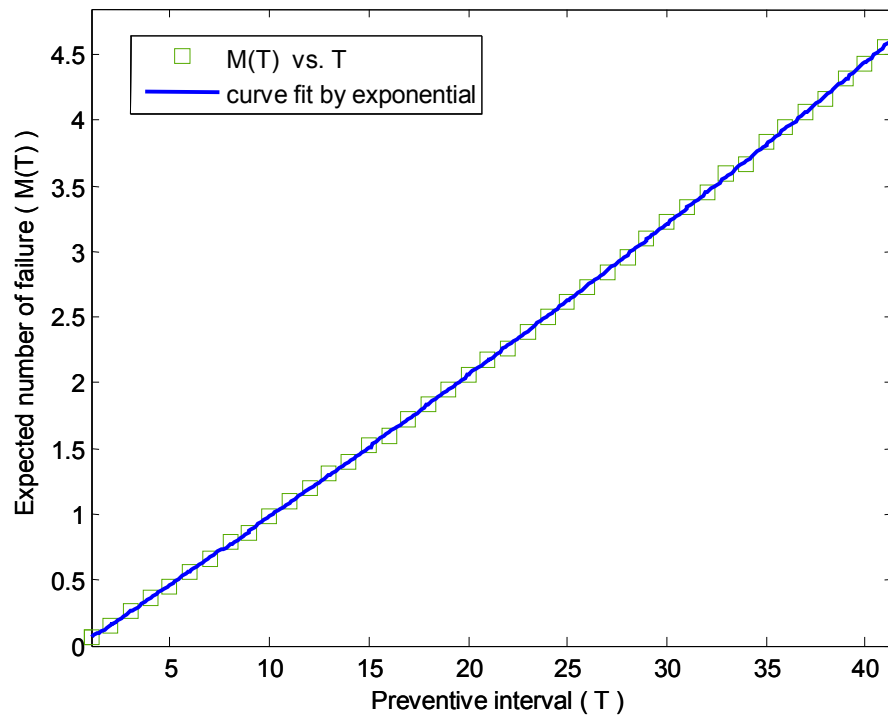
Figure 21: Renewal Function Curve Fit for Analysis 4 with Repair Time

The equation (39) shows the function of fitted curve.

$$y = 6.736 \exp(0.01005x) - 6.779 \exp(-0.004648x) \qquad (39)$$

The curve is very close to linear function, but it does have curvature from exponential functions. It can be observed that at time 0, the expected number of failure is not 0, and it is negative number for the region very closed to zero. It should be noted that the purpose of curve fitting, or more generally constructing RSE, is to have continuous functions from the discrete simulation results. The curve fit value near zero is nonsense, and the simulation results at this region may result in an incorrect fitted curve. Nevertheless, it is assumed that the bias from simulation results is considerably small from the validation from Analysis 1, and the optimal solution is expected lie between 0

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and 40. The justification of the preset range for MC simulation follows a same reasoning form the example of neglecting repair time. The resulted R-square and SSE are reasonable to accept the curve.

Next, availability up to given Preventive interval is fitted in Figure 22.
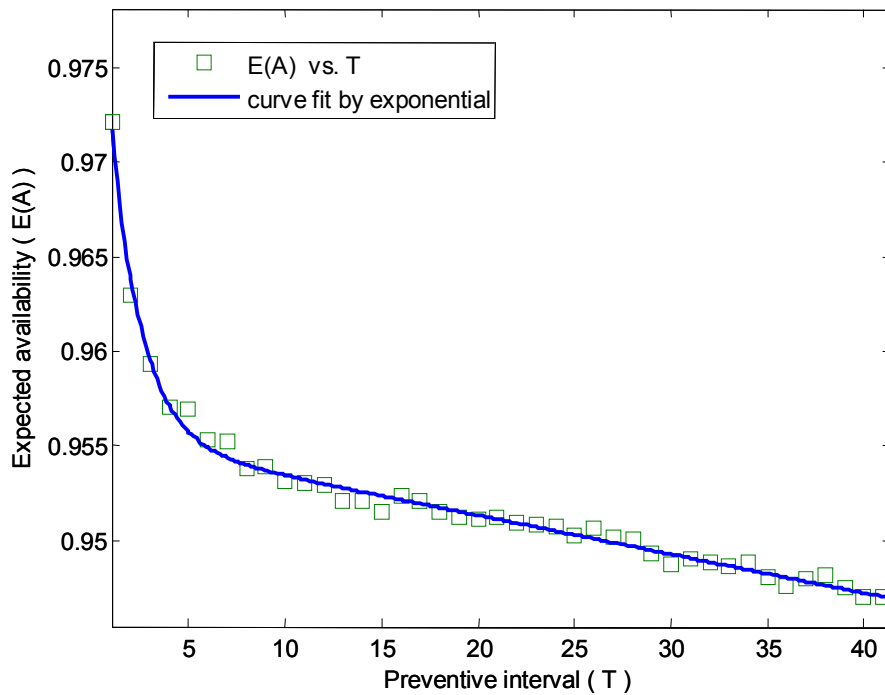
Figure 22: Availability Curve Fit for Example with Repair Time

The curve fit function is shown in equation (40) with a combination of exponential distributions.

$$y = 0.03109 \exp(-0.6317x) + 0.9555 \exp(-0.0002165x) \qquad (40)$$

Both renewal function and availability have monotonicity under the prescribed range of interval. The preset interval is important since the availability would decrease much faster as PM interval increases. That is why combination of exponential

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distributions is selected over polynomial function for this example to represent the sharp decrease in availability. It should be noted that the equation (40) is only valid for the given DOE table range from 0 to 40. For example, If the preventive interval is 245 time unit then it would expect more than 30 failures happened during the period. The quasi-renewal parameter α of 0.95 after 30 failures gives $0.95^{30} = 0.21$ of the first expected lifetime. The corresponding availability value from equation (40) would be much higher value for interval 245 than the exact availability. This example illustrates how fast expected lifetime decrease as number of failure is cumulated, and why the representative formula from regression analysis should be used within the range of DOE table.

Next sections discuss how the optimized periodic PM interval varies with the combination of CM and PM cost with the availability constraint. It will be observed that availability constraint can be active or inactive based on the given cost settings.

5.4.2.1 CM Cost is $4500 and PM Cost is $500 With 0.95 Availability

Same as in the previous example without repair time, the response of objective function, long run average cost rate, is plotted with respect to PM interval in Figure 23. The difference from the previous example is that repair time is included, and availability is set as a constraint to search for the optimal PM interval.
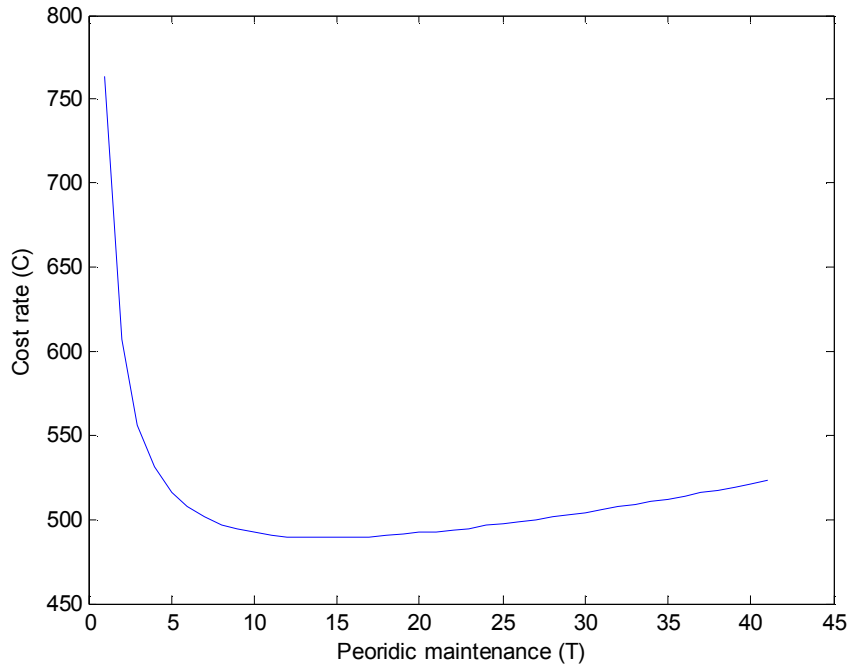
Figure 23: Simulated Result for Analysis 4 with Repair Time (Cost Setting1)

The optimal result is tabulated in Table 11.

Table 11: Result for Analysis 4 with Repair Time (Cost Setting 1)

Optimal PM interval	Cost rate	Availability
14.227	487.12	0.9526

As one can see optimal PM interval is reached with marginal availability.

5.4.2.2 CM cost is $4500 and PM cost is $3000 with 0.95 Availability

The cost setting indicates that PM cost is increased to see how this can affect the optimization with the availability constraint. It is observed from the previous example that relative higher PM cost tends to increase the PM optimal interval. The simulated objective function is plotted in Figure 24 and the result with shown in Table 11.

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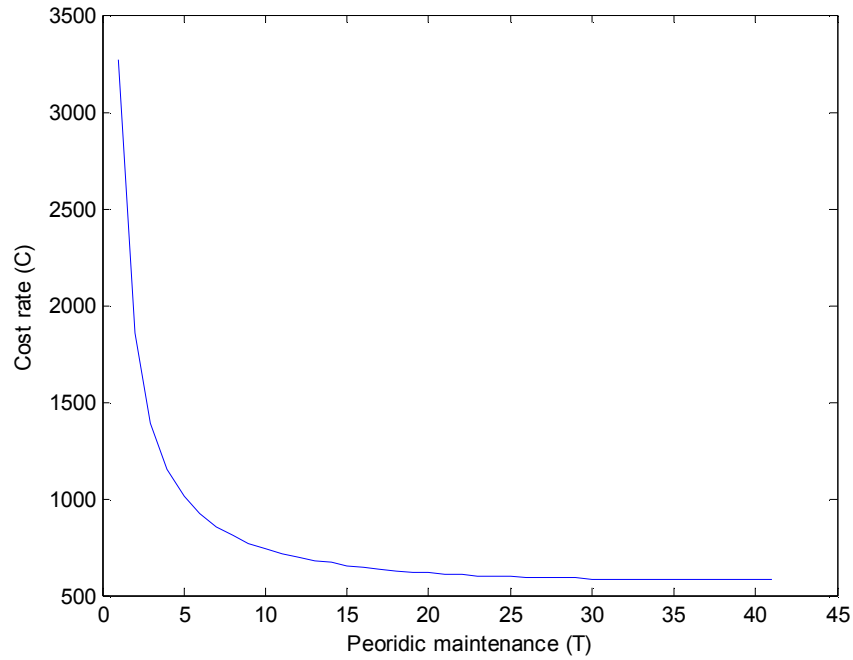
Figure 24: Simulated Result for Analysis 4 with Repair Time (Cost Setting 2))

Table 12: Result for Analysis 4 with Repair Time (Cost Setting 2)

	PM interval	Cost rate	Availability
With constraint	26.66	593.2584	0.95
Without constraint	37	583.2611	0.9478

It is observed from the result that increase of PM cost forces optimal PM interval to be increased, but the PM interval is set by the availability constraint.

The Analysis 4 can be extended to include more components in the model by applying the structure from Analysis 1. Then, it would find the optimal PM interval under the predetermined system availability. Even though individual reliability is high, the system reliability would be low from the arrangement of components. As the proverb quotes "A chain is only as strong as its weakest link", the most important component in the series system is the weakest component. The concept can be applied to availability

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that the one with longest repair time or shortest lifetime will influence the system availability. The phenomenon can be readdressed in Analysis 5 which deals with a maintenance problem for series system.

Another comment on the result is that opportunistic maintenance, which addresses the advantage of joint maintenance, would require more than one decision variable depending on the type of policy. An example of implementing opportunistic maintenance is covered in Analysis 6 by parallel system. Moreover, the opportunistic maintenance is applied to FADEC system to capture any cost saving during join maintenance. More detail explanation and implementation of the opportunistic maintenance are examined in Chapter 6 with FADEC system.

In summary, the validation of utilizing MC simulation and regression analysis in the proposed methodology for the single component maintenance problem promise the applicability to real-world applications, where the system is consist of multiple components which lifetime distribution may be arbitrary not to have easier mathematical representation. The Analysis 4 also provides the flexibility of the proposed methodology, since the main framework of MC simulation and regression analysis is same for both cases. The current mathematical approach for this problem has different assumptions based on the repair condition to increase the mathematical tractability. Of course, the derivation processes involved in the current mathematical approach are different for both cases.

5.5 *Analysis 5: Optimal Maintenance for the Series System*

This section studies one of the common multi-unit systems, i.e. series system, to obtain the availability and optimal maintenance policies. It is anticipated that there are multiple decision variables involved in the optimal maintenance problem.

5.5.1 Problem Description for Series System

The series system consists of 4 components, and the schematic diagram to represent the state of each component can be illustrated as follows.
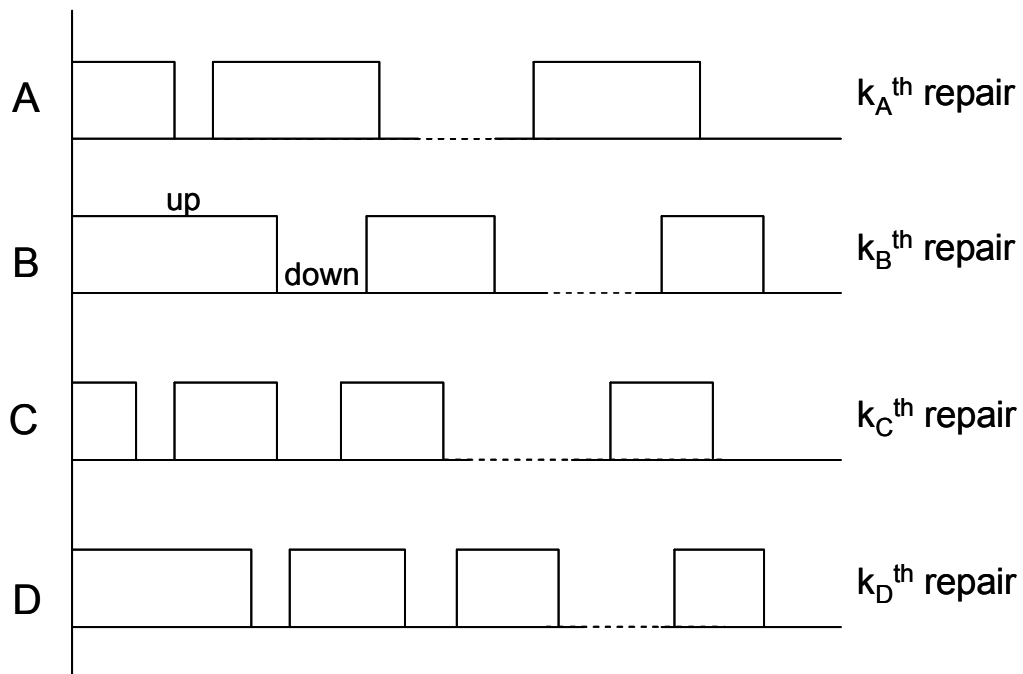

Figure 25: Schematic Diagram of States of Components in Series System

The schematic diagram represents components' states up to $k_i^{\text{th}}, i = A, ..., D$ repair at which perfect repair is performed. Therefore, each component would subject to $k_i - 1$ imperfect maintenance. Moreover, based on the combination of k_i values, the long-run

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average cost rate and availability will have different outcomes. For instance, if all k_i values are 1 then this series system is subject to the perfect maintenance whenever there is component's failure. Typically, perfect maintenance is more expensive than the imperfect maintenance, because replacing the whole component tends to cost more than repairing the component to extend its life.

Whether the repair is imperfect or perfect, the suspended animation is assumed in Analysis 5. It is reasonable assumption that failure of each component results in the failure of system, and the relatively short repair time prevents the lifetime of other component from further deterioration during this period. Next paragraph discusses about how current mathematical approaches can solve the optimal maintenance problem, and what could be the limitations from the approach. The component parameters are also given to compare the results from the current availability mathematical approach with the one from the proposed methodology.

Current mathematical approaches provide mathematical representation of long run average cost rate and availability with respect to the decision variables addressed above. As mentioned from previous chapters, the approaches involve intensive mathematical derivations on reliability measures, and the process requires assumptions on lifetime and repair time distribution to increase the mathematical tractability. Barlow and Proschan (1975) derived mathematical formulas for availability under perfect maintenance. [5]. More recently, Wang and Pham (2006) extended the model to capture the imperfect maintenance by quasi-renewal processes [101] with the correlation defined in [37]. Nevertheless, these approaches have a common assumption on the distribution to satisfy the sufficient condition for the existence of limiting value. Generalized distribution is

also studied by Zhao (1994), but the work also has assumptions to ensure the existence of mathematical derivation [108].

Table 13: Component Parameters for Analysis 5

Component	Mean time to failure	Mean time to repair	k_i	α_i	β_i
A	50	0.1	6	0.90	1.05
B	100	0.2	5	0.90	1.05
C	1000	1.0	6	0.95	1.05
D	10,000	20.0	7	0.92	1.05

Numerical example is established based on the component parameters listed in Table 13. The exponential distribution for each component guarantees the existence of mathematical derivations for reliability measures under either perfect or imperfect maintenance. Perfect maintenance can be analyzed by setting all of k_i values equal to 1 or imperfect maintenance parameters (α, β) as 1. Mathematical equation for availability by Wang and Pham modeled imperfect maintenance by introducing α, β parameters as listed in Table 13. It is considered not to show the resulting equations for both authors, because it may distract readers from various probability theories used in derivations with many notations from component states and orders of repairs.

Since the set up for component parameters in Table 13 can be viewed as the one input case for MC simulation in the proposed methodology, it is good time to check the accuracy of MC simulation with the analytical result from mathematical approaches. There is, however, a problem with obtaining the limiting values, such as long-run average availability or long-run average number of failure per hour, by MC simulation. The mathematical derivation utilizes probability theories, such as strong law of large number

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and elementary renewal theory, to restate the limiting values in finite terms

One way to simulate limiting (steady state) value by MC simulation is to run for a long time interval. If the simulation duration is long enough then the influence from transient behaviors would be diminished to result in the steady state value for the given measure. The longest mean lifetime from Table 13 is 10,000 hours so the simulation duration is set as 20 times of this value. Without considering imperfect maintenance and repair time, there are about 20 failures from this component during the simulation period. If imperfect maintenance of $\alpha = 0.92$ is modeled then the failure from this item would occur more than 20 times, which assures that the simulation can reach to the steady state. The results are compared in Table 14 for perfect maintenance case.

Table 14: Availability Measures for Series System under Perfect Repair

Availability measures	Barlow-Proschan	Proposed
Limiting availability	0.993	0.993
Limiting unavailability from each component		
$D_{av,A}$	0.002	0.002
$D_{av,B}$	0.002	0.002
$D_{av,C}$	0.001	0.00098
$D_{av,D}$	0.002	0.002
Limiting # of failure from each component		
N_A	0.02	0.0198
N_B	0.01	0.0099
N_C	0.001	0.00099
N_D	0.0001	0.0001

MC simulation in the proposed methodology is performed for 10,000 sample paths with each having simulation duration of 200,000 hours. The differences between simulated result and the analytical solution are very small to be neglected. The same MC simulation scheme is used to generate the case for imperfect repair as in Table 15.

Table 15: Availability Measures for Series System under Imperfect Repair

Availability measures	Wang-Pham	Proposed
Limiting availability	0.9903	0.9903
Limiting unavailability from each component		
$D_{av,A}$	0.0029	0.0029
$D_{av,B}$	0.0027	0.0027
$D_{av,C}$	0.0013	0.0013
$D_{av,D}$	0.0029	0.0029
Limiting # of failure from each component		
N_A	0.0254	0.0254
N_B	0.0121	0.0121
N_C	0.0011	0.0011
N_D	0.0001	0.000124

It can be observed that limiting availability has been decreased for imperfect maintenance case because of increase in downtime and decrease in uptime by quasi-renewal factors. Minor differences suggest that MC simulation in the proposed methodology provides reasonable simulated results, and it is validated to construct RSEs based on the MC simulation results for different combination of decision variables from DOE table to reconstruct the objective function, cost rate, in the optimal maintenance problem.

5.5.2 Optimal Maintenance Problem for Series System

It is required to have the optimal maintenance problem for the series system above. As mentioned in Chapter 2, the long-run average cost rate is set as the objective function. The first step to implement the proposed methodology to the problem is to set up the DOE table for MC simulation.

There are 4 decision variables (k_i s), and it is natural to assume that these variables are bound by certain number. If $k_i = 10$ then α of 0.9 would result in the expected lifetime of 9th imperfect maintenance be $0.9^9 = 0.38$, i.e. 38 % of the expected lifetime of the initial distribution. This indicates that $k_i = 10$ setting is very extreme case which yields too many repairs which increase the total maintenance cost. Based on the simple reasoning by reduction in expected lifetime from renewal factors, the bound for each decision variables are set as $1 \le k_i \le 5$. This bound may be changed for the imperfect and perfect cost setting for each component, but it is found out that the bound is sufficiently large to include the optimal combination of decision variables to yield the minimum cost rate. Another consideration for DOE table comes from the integer values of decision variables. Unlike other examples from the previous analyses, the solution should be integer. The standard DOE table of 3-factor Central Composite Design Inscribed (CCDI) is used in this example. CCD is selected over Box-Behnken from more samples for the center point. Moreover, inscribed design is chosen to generate the interior points from the boundary of CCD design [17]. Commercial software, JMP, is used to generate the CCDI table, and the total number of combination for decision variables results in 31. Next step is to run each row of DOE table with MC simulation.

After running MC simulation, we can generate the table consists of input

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combination of k_is and the corresponding simulation results for reliability measures, such as expected number of repair, renewal cycle, etc. RSEs are constructed for each reliability measure based on the discrete simulation results to reconstruct the objective function as follows.

$$\min : C = \frac{\sum_i c_i N_i}{R}$$
$$c_i = \begin{bmatrix} c_i^{imp} & c_i^{per} \end{bmatrix}, c_i^{imp} < c_i^{per}, i = A,...,D \qquad (41)$$
$$N_i = \begin{bmatrix} N_i^{imp} \\ N_i^{per} \end{bmatrix}, i = A,...,D$$

c_i is the cost setting for each component, and it is reasonable to assume that imperfect maintenance cost is lower than that of perfect maintenance (replacement of whole component). N_i denotes the expected number of imperfect and perfect maintenance for the given component. The numerator of equation (41) simply states the total maintenance cost up to a given interval, i.e., R. The given interval R should be infinity by the definition of the limiting cost rate. As mentioned in Chapter 2, if one can find the finite renewal cycle then this cycle can be used as the interval by key renewal theory. That is why many optimal maintenance problems set a system PM interval as one of decision variable to restore the system to the original state. Unfortunately, this series system problem does not have a system PM interval as a decision variable, and it is not trivial to find the renewal cycle solely from the decision variables given, i.e. k_is. The interval, R, is set as a considerably large number as in the MC simulation to demonstrate the steady state behavior of cost rate. Considerably large number of 200,000 hour is validated from availability simulation results in Table 14 and Table 15. RSE for expected

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number of imperfect maintenance of component B is expressed in equation (42) as an example.

$$N_B^{imp} = 308.83 - 0.41\,k_A + 390.13\,k_B + 0.48\,(k_A - 3)\,(k_B - 3)$$
$$+6.69\,(k_A - 3)(k_A - 3) - 117.1\,(k_B - 3)(k_B - 3) \tag{42}$$

The quadratic term by decision variable, k_B, explains abut 97% of the explanatory power. Decision variable, k_A, is included to have interaction term and quadratic term in RSE construction, since either of k_C or k_D would have small influence over the expected number of maintenance of component B. The small influence comes from the relative longer lifetime for component C and D, so their k_i values would not make big difference over the response of component B.

The objective function is reconstructed from the RSEs for reliability measures, and traditional line search algorithm of SQP is utilized to find the optimal solution which is real numbered. Then, one can find the nearest integer combination to search for the optimal integer solution. Ten combinations of decision variables are selected from the continuous solution, and for MC simulation on these combinations results in the minimum cost rate as \$0.17849 having decision variable combination of $[k_A\quad k_B\quad k_C\quad k_D] = [4\quad 3\quad 3\quad 5]$. The result is consistent with the outcome from the mathematical approach by Wang and Pham (2006) [101].

5.6   *Analysis 6: Optimal Maintenance for the Parallel System*

The parallel system is used more often than the series system because of redundancy in reliability. That is the reason why most of the practical applications consist of family of parallel systems which include cold or hot standby systems. As mentioned in series system example, it is not optimal to perform perfect repair (replacement) whenever there is a failed component. Like imperfect maintenance is provided based on the k_i for the series system, minimal repair is performed for the parallel system. The mathematical definition of minimal repair is stated in equation (1) to indicate that the action of maintenance does not improve the reliability of the system. The reason for performing a minimal repair in the parallel system is based on the fact that there are still redundant components working to provide system's operating status. Moreover, it is optimal to perform minimal repair for the parallel system under certain maintenance policies [50]. Following section discusses about how (τ, T) policy can capture the opportunistic maintenance for economic dependencies as well as the benefit from minimal repair.

5.6.1   Problem Description for Parallel System

Wang and Pham (2000) proposed (τ, T) policy for the parallel system as shown in Figure 26 [102].
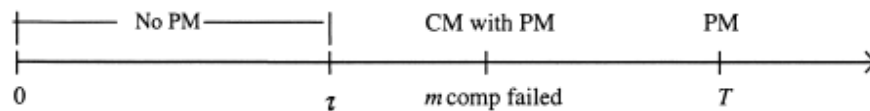

Figure 26: (τ, T) Maintenance Policy for Parallel System

The policy suggests that there will be minimal repair before time τ, since the component is young enough. The successive minimal repair before time τ increases the failure rate for the component as time passes. After time τ has been passed, the operator counts the number of failing component. If the number of failed components reaches a predetermined integer value m then the failed m components are perfectly repaired, and the whole system undergoes preventive maintenance to restore the system as good as new. There may be the cases that the number of failed components does not reach m before time T. Under this condition, perfect PM on system is performed to make the process to repeat as in the renewal process.

As one can see form the (τ, T) policy, the policy includes economic dependency during the period of $\tau \leq t \leq T$. The relatively cheaper cost of performing CM with PM together than performing separately can result in the need for the opportunistic maintenance to reduce the total cost rate while satisfying required availability. The concept of an opportunistic maintenance has been studied in various literatures ranging from 2-unit system to multi-unit system [11, 43, 80, 109]. It can be expected that if opportunistic maintenance is not optimal for a given setting then τ would be equal as T to avoid any chance of the opportunistic maintenance. If it turns out that the opportunistic maintenance is always optimal for a given setting then T would be considerably large number. In summary, if there is a strong economic dependency for a multi-unit system then joint maintenance, such as OM, should be considered as in (τ, T) policy.

The current availability mathematical approaches derive the long-run average cost rate and long-run average availability to construct optimal maintenance problem. As one can see from Figure 26, the general idea of renewal cycle can be easily understood from

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the maintenance policy. Nevertheless, the mathematical derivations for the renewal cycle or any other reliability measures with respect to τ, T are not trivial. The resulting survival function after the period (τ) requires knowledge about the order statistics of successive survival function after the minimal repair. Moreover, the derived equation itself should be solved by numerical schemes or approximations from complex integral equations. The contribution by Wang and Pham (2000) is remarkable by generalizing other maintenance policies by (τ, T) maintenance policy, but the process should be flexible and easy to be practical.

5.6.2   Optimal Maintenance Problem for Parallel System

The numerical example from Wang and Pham (2000) is addressed here to compare the result from analytical approach and the one from the proposed methodology. A 2-out-of-3 aircraft engine system is studied, and each engine is assumed to follow Weibull distribution. The optimal maintenance problem can be expressed by minimizing the long run average cost rate as follows.

$$\min: \ C = \frac{c_m N_m + c_{om} N_{om} + c_{pm} N_{pm}}{D}$$

c_m : minimal repair cost, N_m : expected number of minimal repair
c_{om} : opportunistic maintenance cost, N_{om} : expected number of OM   (43)
c_{pm} : preventive maintenance cost, N_{pm} : expected number of PM for system
D : renewal cycle

If the minimal repair cost only has deterministic term then the first term in numerator can be derived easily. Sheu (1992) considered probabilistic (random) minimal cost by taking expectation with respect to random variables, such as number of minimal

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repair and random term in the minimal repair [88]. Based on the property of NHPP mentioned in Chapter 2, the derivation of total minimal cost up to time τ can be mathematically derived. Remaining reliability measures in equation (43), i.e. the expected number of OM, the expected number of system PM cost and system renewal cycle, are not trivial to be obtained by mathematical derivations. That is the reason why MC simulation and RSE construction in the proposed methodology are utilized.

The decision variables of τ and T are different from the decision variables, k_is, in the series system. The integer values of k_is have no regulation but to be bounded by 5 from previous analysis. On the other hand, there is a regulation in (τ, T) policy that τ can not be greater than T. If τ is greater than T then expected number of opportunistic maintenance would be always zero to have a trivial outcome. It is recommended not to waste the number of samples by using the traditional DOE tables such as CCD or Box-Behnken. A customized DOE table is generated based on the constraint in decision variables to capture the information about reliability measures, such as D, N_{om} and N_{pm} in equation (43). Here two customized DOE tables are generated by different ranges. First DOE table covers wide range of decision variables. The wide range assures the optimal solution would lie within the range from the upper boundary selection by the rule of thumb mentioned in previous examples. The upper bound of T is set as 3 to 4 time of expected lifetime of component, i.e. 440 days. This upper bound is sufficiently large for the 2-out-of-3 system, for which failure of 2 components would result in system failure. To create the DOE table for the analysis, one need to first assign discrete sample points for decision variable, T, between 50 to 1300. The lower bound is selected to have any smaller number compare to the expected lifetime of a component. Inner point DOE table,

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such as Latin Hyper Cube, is used for assigning the T for the DOE table. τ is assigned between the T values to capture the different intervals for OM. The resulting DOE table has 55 combination of τ and T. The RSE of expected number of opportunistic maintenance is shown as example, and resulting optimal policy and figure of merit is tabulated in Table 16.

Table 16: Result for the Customized DOE table 1 for (τ, T) Policy

DOE table 1	
Range	$50 \leq \tau \leq 1200, \ 50 \leq T \leq 1300$
N_{om}	$N_{om} = -0.078 - 0.044\tau + 0.004T$ $+8.01 \times 10^{-5}\tau T - 4.86 \times 10^{-5}\tau^2 - 3.17 \times 10^{-5}T^2$ $+2.96 \times 10^{-9}\tau^3 - 2.77 \times 10^{-9}T^3$
(τ, T)	$(309.98, 409.98)$
Cost rate	0.1798

The result from mathematical approach yields $(\tau, T) = (335.32, 383.99)$ with having a long-run average cost rate of 0.1826. The error of 1.5% by customized DOE table 1 is acceptable, if the objective function value is compared. The deviation of optimal maintenance policy is not small enough to be neglected. It is concluded that the predictability of given RSEs from customized DOE table 1 is not sufficient even with the polynomial order of 3 to increase the explanatory power. The goodness of fit measure from the polynomial fit by RSEs for the wide range suggests the regression result is not satisfied by large SSE. Following paragraphs examine how this problem can be solved if the narrow range is selected for RSEs construction for reliability measures.

Results from the customized DOE table 1 suggest that there would be the optimal solution in the range of 300 and 400. The customized DOE table 2 is generated solely for

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this range to check any improvements on the results.

Table 17: Result for the Customized DOE table 2 for (τ, T) Policy

DOE table 2	
Range	$300 \le \tau \le 400, \ 300 \le T \le 400$
N_{om}	$N_{om} = -0.106 - 0.038\tau + 0.004T$ $+5.64 \times 10^{-5} \tau T - 3.44 \times 10^{-5} \tau^2 - 2.25 \times 10^{-5} T^2$
(τ, T)	$(329.23, \ 392.58)$
Cost rate	0.1826

The narrower range for RSE construction provides simple quadratic RSE to fit easily without any higher order term or any transformation. Moreover, the goodness of fit measure is acceptable. As a result, the optimal maintenance policy is very close to the result from analytical solution, and the cost rate is exactly same.

The procedure above can be applied sequentially over the initial range form 50 to 1300. For example, one can start with the narrow range of 50 to 200 for RSE construction and move the interval windows to cover the entire range of initial DOE table. It should be noted that the reason for having several windows for initial (wide) DOE table for regression analysis is to avoid the situation when a single RSE over the wide range of decision variables has poor goodness of fit result. Therefore, narrower ranges are selected for the multiple regression analyses to construct the better RSEs.

The process mentioned above assures the total number of sample runs for MC simulation does not have to be changed, since the multiple regression analyses are performed on the simulation results MC simulation under wide range of DOE table. Generally, the process time for RSE module takes less time than the time for MC simulation module, so multiple regression analyses would be practical in terms of

rapidity of process.

In summary, the proposed methodology is performed for the parallel system under (τ, T) policy because the mathematical derivation to construct the optimal maintenance problems is not trivial. Moreover, the closed-form expression of a long-run average cost rate tends to include integral equations which should be numerically solved for most of the IFR distributions. The MC simulation in the proposed methodology does not require any assumptions on the problem, and the surrogate models (RSEs) have much simpler formulas than the results from analytical approaches.

It is also suggested that if the goodness of fit test turns out to be poor in RSE construction for an initial range of DOE table then multiple regression analyses can be performed on the narrower ranges of the decision variables. The resulting RSEs from the narrow range of decision variables would have good fit by simple quadratic equations. These surrogate models reconstruct the objective function in the optimization problem to yield the optimal policy for the given interval. The local optimal solution from each narrower interval is compared to have global minimum cost rate.

5.7 *Analysis 7: Optimal Maintenance Considering Economic Dependency*

The parallel system from the previous analysis already considered an opportunistic maintenance to capture the economic dependency. In this section, another form of opportunistic maintenance is introduced to the system consists of the structure of parallel and series together. There is no benchmark analytical solution for the arbitrary system like this, but it would be addressed that how MC simulation and RSE construction in the proposed methodology can easily adapt to such an arbitrary system to achieve the optimal maintenance policies for given maintenance cost settings. It would be also discussed in this chapter that a customized DOE table over the standard DOE table is recommended for the optimal maintenance problem having multiple decision variables if there are constraints in decision variables.

5.7.1 Problem Description for Analysis 7

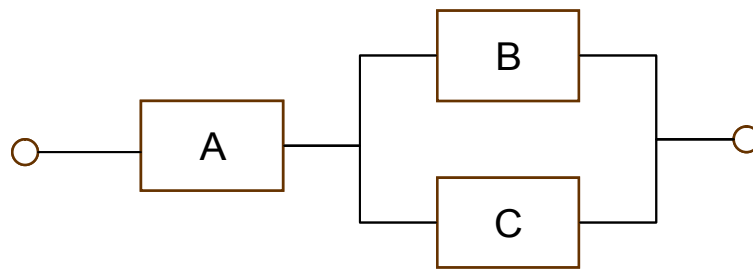A simple multi-unit system is considered as in Figure 27.

Figure 27: RBD for the Multi-unit System for Analysis 7

The main purpose of this analysis is to get familiar with the system maintenance with economic dependency. Modeling the economic dependency is repeated in Chapter 6 with more complicate structure by redundant components.

Table 18: Parameters for Analysis 7

Component	Distribution
A	Weibull distribution (0.006,2)
B	Exponential distribution (0.008)
C	Exponential distribution (0.005)
Repair	Exponential distribution (0.2)
Opportunistic Repair	Exponential distribution (0.15)

As one can observe from the table above, the component A can be considered as the critical component, since it has the shortest expected lifetime with strictly IFR distribution. The series arrangement of component A can make the system reliability more depends on the status of the component A. Generally, CM cost for all components in the system should be also considered when the critical component is to be determined.

The expected opportunistic maintenance time in Table 18 is assigned as smaller than the individual repair time to increase the benefit of the join maintenance.

If component A is to set as the critical component then the operators want to check the status of component A whenever other components are in CM. For instance, after component B has been failed, the operators want to check the component A even if it is working at that time. The strictly IFR distribution of component A would result in more chance of failures as time passes. In such circumstance, it may be optimal for operators to decide whether to perform PM for component A or not, while executing CM for the failed component. The good way to set the go / no-go criterion for this type of opportunistic maintenance is to introduce new decision variable like a trigger age. The general description of the trigger age is explained in Chapter 6. The simple understanding of the trigger age is that the trigger age is compared with age of A to decide the initiation

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of opportunistic maintenance. There are two trigger ages from component B and C under the problem set up like Analysis 7. As same as the previous analyses, PM interval for component A is also considered as a decision variable. If we have decision variables as $[X_A \quad X_B \quad X_C] = [100 \quad 20 \quad 30]$ then there would be PM on A after 100 time unit. This setting also indicates that the age of component A is compared with the predetermined 20 whenever component B is in CM. If age of A is greater than 20 then opportunistic maintenance is performed to make the system as good as new. Since exponential distributions are used for component B and C, PM for component A would make the system as good as new by the memoryless property of exponential distribution. The same argument can be applied to the case of component C failure.

5.7.2 Optimal Maintenance Problems for Analysis 7

The 3 decision variables are to be determined by the optimal maintenance problem, and the objective function can be written as below.

$$L = \frac{c_{fA}A + c_B B + c_{BA}BA + c_C C + c_{CA}CA + c_{pA}}{D}$$

$$c_{BA} = \alpha \times (c_{pA} + c_B), \ \alpha < 1, \ c_{CA} = \beta \times (c_{pA} + c_C), \ \beta < 1$$

A : Expected number of CM of component A during renewal cycle (D)

B, C : Expected number of CM of component B, C during renewal cycle (D) (44)

BA, CA : Expected number of OM of (B,A) and (C,A) during renewal cycle (D)

c_{fA}, c_{pA} : CM and PM cost of component A

c_B, c_C : CM cost for component B and C

c_{BA}, c_{CA} : opportunistic maintenance cost of (B,A) and (C,A)

The ratio of opportunistic maintenance, i.e. (α, β), in equation (44) is set as 70% to assume that there is 30% reduction from OM. The 5 reliability measures in equation

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(44) are obtained from MC simulation and RSEs construction. The proper DOE table is to be used for the given problem, since we know that the both trigger ages larger than the PM interval of component A does not have a physical meaning. The PM interval (X_A) sets the maximum of the renewal cycle from the maintenance policy described above. If trigger ages from component B or C have the value above the renewal cycle then there would be no opportunistic maintenance. Moreover, it should be considered that trigger ages from component B and C are compared with the age of component A, not with the operation time recorded from the initial state. The age of component A renewals whenever there is CM for the component, and it is expected that the trigger age would be somewhat bounded by the expected life time of component A which is around 125 hours from equation (11). The following paragraph compares the results from the different types of DOE tables utilized in MC simulation module.

Box-Behnken and CCDI 3-factor DOE tables are considered as the standard DOE tables, and customized DOE table is constructed to capture the constraint in the decision variables like the DOE table used in the parallel system.

Table 19: DOE comparison for Analysis 7

	Box-Behnken	CCDI	Customized
Sample points	15	16	63
Error dist.	(0.75%, 0.70%)	(0.75%, 0.67%)	(3.71%, 4.51%)
Pred. error dist.	(20.67%, 25.20%)	(29.27%, 17.28%)	(2.41%, 2.79%)
(X_A, X_B, X_C)	(351,289,297)	(288,288,141)	(279,113,105)
Cost rate	18.36	18.32	18.02

The standard 3-factor DOE tables require less sample points to construct quadratic equation. For example, it only requires 3 points to be fitted by the univariate quadratic equation, since the curve only has 3 unknown parameters. Generally, if the unknown function is not exactly a univariate quadratic function then it may require more points. The regression analysis, such as standard least square method, can be applied to the sample points to construct the quadratic curve that can capture the responses of the discrete simulation as much as possible. The same approach is performed here, since it is considered that the reliability measures are not exactly quadratic function, but they are assumed to be represented by the quadratic equation. It is observed that using minimum numbers of sample point from Box-Behnken and CCI DOE table can only give the good fit for the in-sample case. The error distribution of in-sample has low mean and standard deviation. If out-of-sample data are fitted with the constructed RSEs then the prediction error is unacceptably high for standard DOE cases. The RSEs constructed from the standard DOE tables should not be used as the representative function to characterize the behavior of the responses.

The customized DOE table requires more sample points because it discretizes the decision variable more than 3 levels. PM interval for component A (X_A) has 7 level and two trigger ages have 3 level to result in 63 total sample points. X_A is more discretized, since it is expected that the reliability measures, such as expected number of failure of the component or the renewal cycle, are more influenced by the PM interval of the component A.

The figure of merit value from Table 19, i.e. cost rate at the optimal policy, seems very close to each other, but the optimal policy is very different. If the optimal policies

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from the standard DOE tables are inputted to the cost rate RSEs by customized DOE table, the resulting cost rate would be higher. Some examples of RSEs constructed from customized DOE table are listed below.

$$A = 0.487 + 0.362X_A' + 0.108X_B' + 0.057X_C' + 0.117X_A'X_B' + 0.07X_A'X_C' + 0.028X_B'X_C'$$
$$- 0.137X_A'^2 - 0.019X_B'^2 - 0.014X_C'^2$$
$$B = 0.47 + 0.133X_A' + 0.342X_B' + 0.055X_C' + 0.107X_A'X_B' + 0.048X_B'X_C' \quad (45)$$

$$X_A' = \frac{X_A - 185}{165}, X_A' = \frac{X_A - 55}{45}, X_A' = \frac{X_C - 60}{40}$$

The RSEs are valid through 0 to 400 region for X_A and 0 to 150 for $X_{B,}X_C$. The narrow range for the trigger age can be understood by the fact that the trigger ages are compared with the age of component A, and the twice of expected lifetime of component A is regarded sufficient enough to find the solution within the region. Equation (45) also implies that component A is considered as the critical component, so full quadratic equation is used to increase the accuracy. These surrogate models (RSEs) are used to reconstruct the objective function, and numerical optimization is preformed with the different cost settings.

Table 20: Result for Analysis 7

Maintenance cost setting		Proposed
$c_{fA} = 1500$ $c_{pA} = 500$	70% OM	$(X_A, X_{B,} X_C) = (279.31, 112.97, 105.56)$ Cost rate $= 18.0219$ Availability$=0.9734$
$c_{fA} = 3500$ $c_{pA} = 50$	70% OM	$(X_A, X_{B,} X_C) = (16.91, 8.83, 0)$ Cost rate $= 10.23$ Availability$=0.9550$

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The CM costs for the component B and C are set as 700 and 900, respectively. The relative small PM cost results in shorter PM interval and trigger age. The figure of merits, such as cost rate or availability, can not be plotted, since there are 3 decision variables. The sensitivity plots are shown below to explain the responses, if two decision variables are fixed at the optimal point.
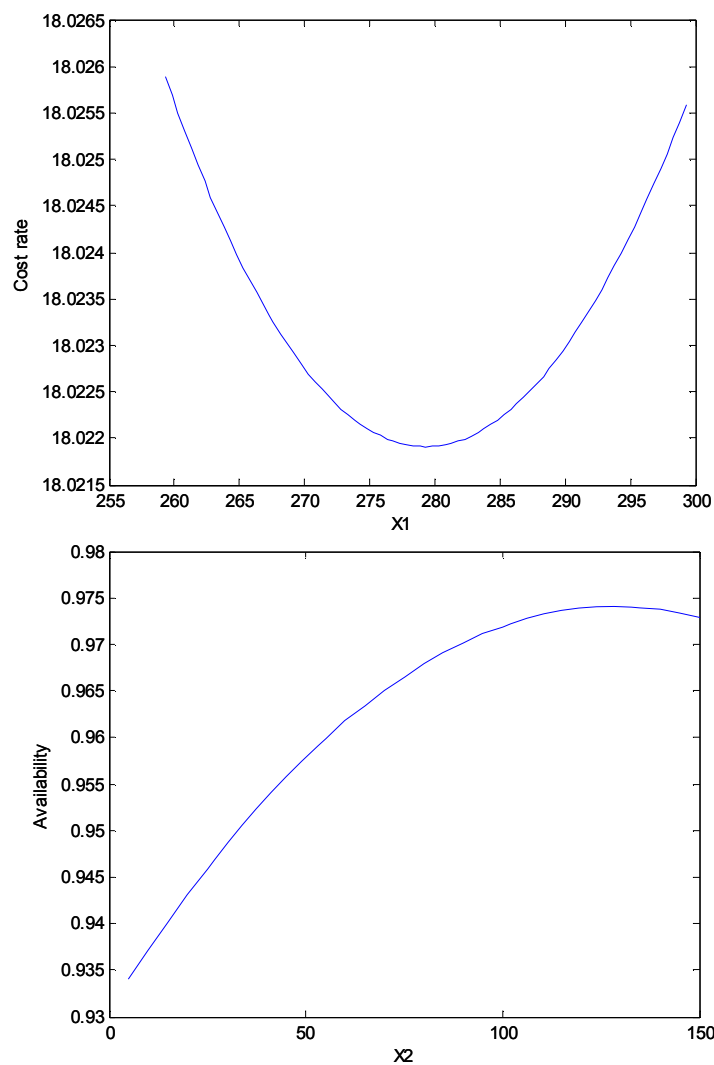

Figure 28: Sensitivity Check for Analysis 7

The upper plot is generated by varying X_A value, and other decision variables are fixed at the optimal setting. As expected, convexity with respect to PM interval is shown in the plot. Higher cost rate in low PM interval is from the frequent maintenance during short renewal cycle, and the increase in cost rate after the optimal point of $X_A = 279.31$ can be explained by the influence of relative large CM cost by strict increase in the expected number of failure of component A.

The lower plot illustrates the sensitive of the availability. Availability has concavity with respect to trigger age from the component B, i.e. X_B. The repair time is considered in this analysis, so increase in repair time in given renewal cycle should reduce the availability. As mentioned before, PM interval is the main driving factor for the renewal cycle, and the PM interval is fixed for availability sensitivity check. The change in renewal cycle due to X_B is relatively small compared to the increase in downtime to result in low availability. The decrease in availability after a certain point in the figure can also be explained by the repair time. As X_B increases, there would be lesser chance of opportunistic maintenance when component B is in repair. The setting results in more CM for the component A, and it is assumed in the problem that OM not only induces the cost saving, but also saves the repair time. Therefore, the availability plot suggest OM is required for the X_B as it grows above the certain value.

5.8   Summary of Preliminary Analyses

Seven preliminary analyses are performed in this chapter. Each of analysis is design to meet the certain goal. Analysis 1 is to show the simulation capability of the proposed method over SPN@ in dealing with a multi-unit system availability simulation under imperfect maintenance. Analysis 2 and Analysis 3 discuss the need for mathematical models for current inspection and replacement policy. Moreover, the mathematical models in the proposed method can give flexible solution while subjective judgment is minimized. Analysis 4 illustrates that how periodic maintenance of a single component under quasi-renewal process can be solved by the proposed methodology without having any assumption on lifetime or repair time. Analysis 5 and Analysis 6 provide the capability of the proposed methodology to search for the optimal maintenance policies under multi-unit system. Opportunistic maintenance is discussed in Analysis 6 and 7 to consider the economic dependencies among the components. The results from the proposed methodology for multi-unit system agree with the analytical results from the current available mathematical approach which involves mathematical derivations.

In conclusion, the overall procedure of MC simulation and RSE construction of the proposed methodology has not been changed through out various examples, whether it is single or multi-unit system. This flexibility of the proposed methodology suggests the broader applicability to real-world applications. Following chapter examines the optimal maintenance problem of FADEC system in detail to set up the optimal maintenance policies under various modeling assumptions.

CHAPTER 6   OPTIMAL MAINTENANCE FOR FADEC

FADEC system has gained popularity from the easiness of control and automation. The tradition engine control system requires the considerable workload from pilot, and maintenance of such a system is not standardized if it is compared with the case of FADEC system. The benefits of FADEC system on piston engine is well described in [91]. Nevertheless, the automation of FADEC system induced failure as in Boeing 777 accident in 2008 [1, 64]. Some researchers studied FADEC system with different types of maintenance to compare the reliability with respect to operational time under Markov model [40], and other researchers proposed more advanced architectures of FADCE system, such as Distributed or Open engine control architecture, to increase the maintainability and flexibility of FADEC system [8, 9]. In this chapter, the generic FADEC system is studied to obtain the overall view of system, since FADEC systems with advanced architectures focus on the properties of modular (component) bases.

The capability of the proposed methodology is further examined by extending to include a multi-component maintenance problem with dependencies. Section 5.2 already discussed how to improve the industry-standard practice for TLD of FADEC system by including the cost distribution for LT component maintenance. In this section, the problem is revisited under the optimal maintenance problem from system point of view.

6.1   Problem Description for TLD of FADEC System

It is mentioned that TLD concept divides components into ST and LT states based on the resulting failure rate of the instantaneous LOTC. The state transitions are shown in

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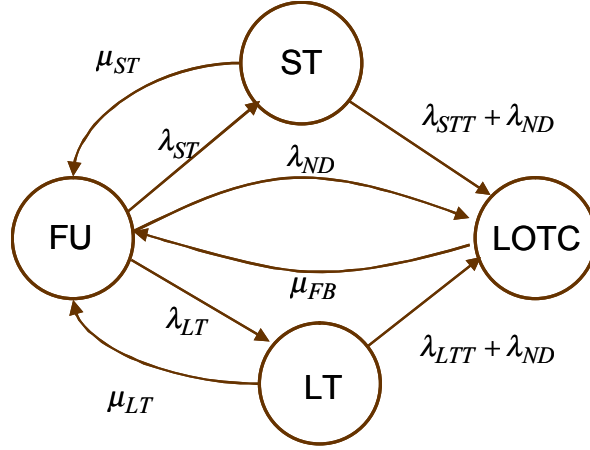Figure 29, and the steady state LOTC rate is expressed in equation (46)

Figure 29: Failure and Repair Rates for TLD of FADEC under Markov Model

$$\lambda_{LOTC} = \frac{\lambda_{ND} + \dfrac{\lambda_{ST}\left(\lambda_{STT} + \lambda_{ND}\right)}{\mu_{ST} + \lambda_{STT} + \lambda_{ND}} + \dfrac{\lambda_{LT}\left(\lambda_{LTT} + \lambda_{ND}\right)}{\mu_{LT} + \lambda_{LTT} + \lambda_{ND}}}{1 + \dfrac{\lambda_{ST}}{\mu_{ST} + \lambda_{STT} + \lambda_{ND}} + \dfrac{\lambda_{LT}}{\mu_{LT} + \lambda_{LTT} + \lambda_{ND}}} \qquad (46)$$

The failure rate, λ_{ND}, represents that of the No Dispatch (ND) component which can lead to total system failure. It is discussed that the shorter dispatch interval for ST component enables the indicator for the repair by means of Minimum Equipment List (MEL) maintenance. Unlike components for ST state, LT components are maintained at periodic intervals under a strategy referred to as the Periodic Inspection/Repair (PIR) maintenance [77]. The appropriate actions based on the different situations are well described in the regulation document such as FAR [34].

In this study, ST dispatch interval is given, and the operators are to decide the LT inspection interval (T_i) from equation (44) that satisfies the regulation on the steady-state LOTC failure rate (λ_{LOTC}) rate of 10 failure per 10^6 hours.

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6.2   *Maintenance Scheduling under Exponential Distribution*

Two different settings of lifetime distribution are considered in this chapter. One is the constant failure rate distribution, and another is the strictly IFR distribution. In this section, exponential distribution is used to compare the result from industrial standard approach with the one from the proposed methodology. It is already studied that lack of cost consideration with subjected engineering judgment on inputs from the practical approach has a room for improvements.

Equation (22) is restated below to check where the subjected engineering judgment can occur during the practical approach.

$$E\left[T_{TSF}\right] = T_i - E(T|T \le T_i)$$
$$= T_i - \frac{\int_0^{T_i} \tau f(\tau)d\tau}{1 - S(T_i)} \tag{22}$$

The LT dispatch interval ($1/\mu_{LT}$) in equation (44) can be solved from given LOTC regulation, failure rates and ST dispatch rate. This LT dispatch interval is set as the time since failure (T_{TSF}) in equation (22). Then, it is up to operators to follow equation (22) to have LT inspection interval or not. If operators realize that there have been changes in maintenance cost settings then they are going to adjust the LT inspection interval from equation (22) based on their experience. It is expected that the resulting inspection interval from subjective decision making to capture maintenance cost settings can result in either infeasible solution or sub-optimal solution.

The proposed methodology restates the above problem in a different way. The conversion of the FADEC problem into the optimal maintenance problem achieves more

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degree of freedom to consider the cost aspect along with searching for the optimal maintenance policies as follows.

$$\min : \frac{\text{maintenance cost per cycle}}{\text{renewal cycle}}$$

$$= \frac{c_{fST}ST + c_{fLT}LT + c_{fLOTC}LOTC + c_i}{D} \tag{47}$$

$$s.t. \text{ availability constraint}$$

In equation (47), four reliability measures appear in the construction of the cost rate. The three terms in the numerator (ST, LT and $LOTC$) indicate the expected number of CM at each state during the renewal cycle (D) which is the denominator. The renewal cycle is assumed to be happen when LT inspection is performed. The assumption of renewal at every LT inspection is not unreasonable, since any maintenances of LT components usually takes longer time than ST maintenance, and overhaul inspection of LT tends to require open the FADEC system. Of course, if the system is in an LOTC state at the time of inspection, then the renewal cycle would be the sum of the inspection interval and the expected repair time at the LOTC state, i.e. $1/\mu_{FB}$, to make the system back to the original state (Full Up state).

The MC simulation and RSE construction are the distinctive steps in the proposed methodology to address the unknown elements of the objective function. Here, the LT inspection interval (T_i) serves as the sole independent variable, taken at discrete time periods as per the applied DOE, and the dependent variables are the results of executing MC simulations at each row of the DOE. Since there is only one decision variable to be considered in this problem, simple set of internal points of predetermined bound is

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sufficient. The predetermined bound for LT inspection interval is chosen by the regulation which forces LT inspection interval not to grow beyond twice of time since failure, $T_{TSF}(=1/\mu_{LT})$. After MC simulation is performed, construction of RSEs for the four reliability measures makes the last step of the surrogate modeling process.

The constraint function in equation (45) can also be modeled in the same manner as described above. The question, however, becomes how availability must be modeled in light of the fact that the regulator only has the knowledge of λ_{LOTC}. Fortunately, there appears to be a useful correlation between λ_{LOTC} and availability through the LT dispatch interval as shown in Figure 30.
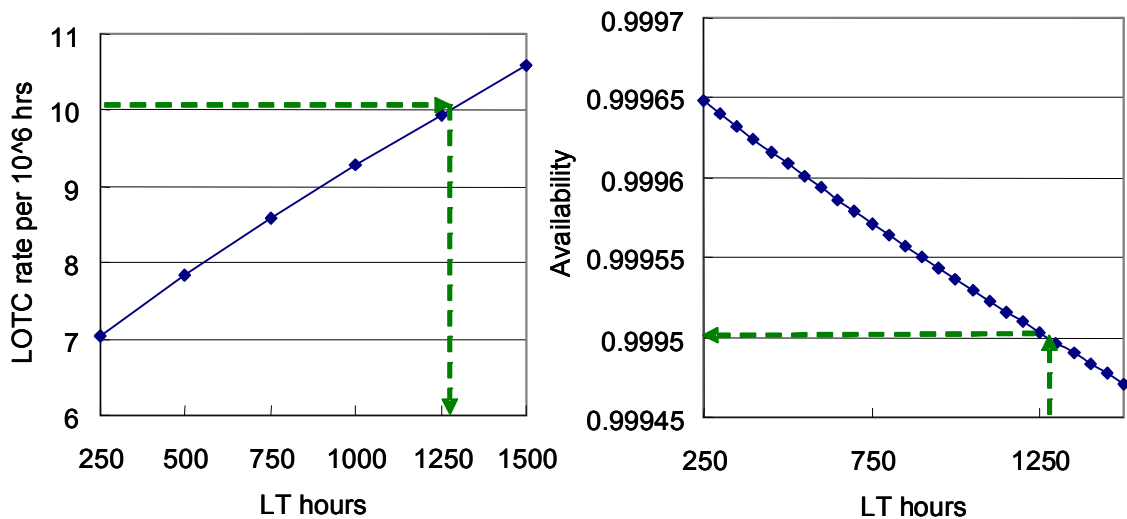
Figure 30: Correlation between LOTC rate and Availability

It is illustrated that λ_{LOTC} of 10 failures per 10^6 hours corresponds to LT dispatch interval of approximately 1255 hours. Moreover, 1255 hours of LT dispatch interval yields availability of 0.9995. Therefore, the knowledge of steady state LOTC rate can be exactly explained by the availability to satisfy the requirement.

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Equation (48) shows the RSEs of the expected number of CM at the LOTC state, as well as availability which were fitted by the function of the LT inspection interval.

$$LOTC = 1.32 \times 10^{-9} \; Ti^2 + 6.66 \times 10^{-6} \; Ti - 4.51 \times 10^{-5}$$
$$\text{Availability} = -3.69 \times 10^{-13} \; Ti^2 - 6.67 \times 10^{-8} \; Ti + 0.9996$$

(48)

The goodness of fit results, depicted in Figure 31, to showcase the predictive capability of both RSEs which were deemed acceptable for the purpose of the present study. The R-square value for the expected number of LOTC repair is close to 1, and the one for the value for availability is considered as acceptable. The wide variation from simulated result for availability suggests increase in MC simulation run. The average relative magnitude of error is indeed very small (order of 10^{-5}), so the fit is considered to be accepted. The discrete points in the plot are the results from MC simulation corresponding to DOE table, and RSEs of a simple quadratic equation are fitted along the points.
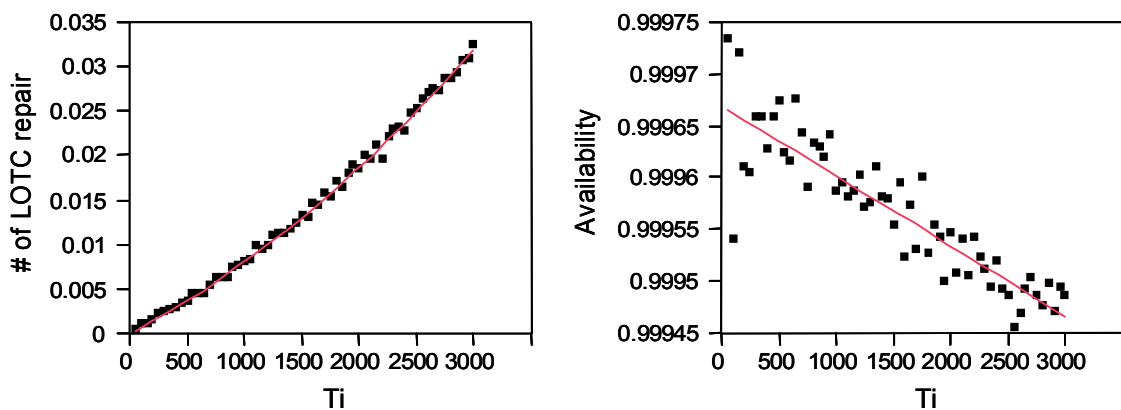
Figure 31: RSEs Fit Plot under Exponential Distribution

The numerical optimization is performed over the optimization problem reconstructed by RSEs, and the results are listed in Table 21 to benchmark the performance of the proposed methodology against that of the traditional approach. The exponential distributions for any transition in Figure 29 are obtained from the ARP-5107 document [20], and the value of CM cost for ST (c_{fST}) and CM cost for LT (c_{fLT}) are fixed at 400 and 800, respectively.

Table 21: Optimization Result under Exponential Distribution

Maintenance cost setting	Industrial approach	Proposed
$c_{fLOTC} = 2500$ $c_i = 100$	$T_i = 2431$	$T_i = 2439$ Cost rate $= 0.1242$ Availability$=0.9995$
$c_{fLOTC} = 10000$ $c_i = 10$	$T_i = 2431$	$T_i = 1053$ Cost rate $= 0.1517$ Availability$=0.99961$

As one can expected, the cost setting is not the influential factor for the industrial approach, since LT inspection interval is solely calculated from equation (22) by setting $E[T_{TSF}] = 1/\mu_{LT}$. As mentioned above, LT dispatch interval ($1/\mu_{LT}$) is obtained by equation (46) to satisfy the requirement. Sometime, the operators realize the importance of cost settings and manipulate the time since failure based on the subjective judgment and historical data. The resulting LT inspection interval may end up with incurring undesirably large cost and may not satisfy the requirement of the steady state LOTC rate.

The LT inspection interval from the proposed methodology is very close to the one from industrial approach, since the availability constraint is active for the given cost setting. It can be observed from Figure 32 that the LT inspection interval is set as the

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twice of LT dispatch interval, if the availability constraint is removed. The reason for maximum LT inspection interval is from the constant failure rate of exponential distribution which may lead trivial solution for PM interval as mentioned in section 5.3. The same cost setting will result in different shape of cost rate for the strictly IFR distribution in following section.
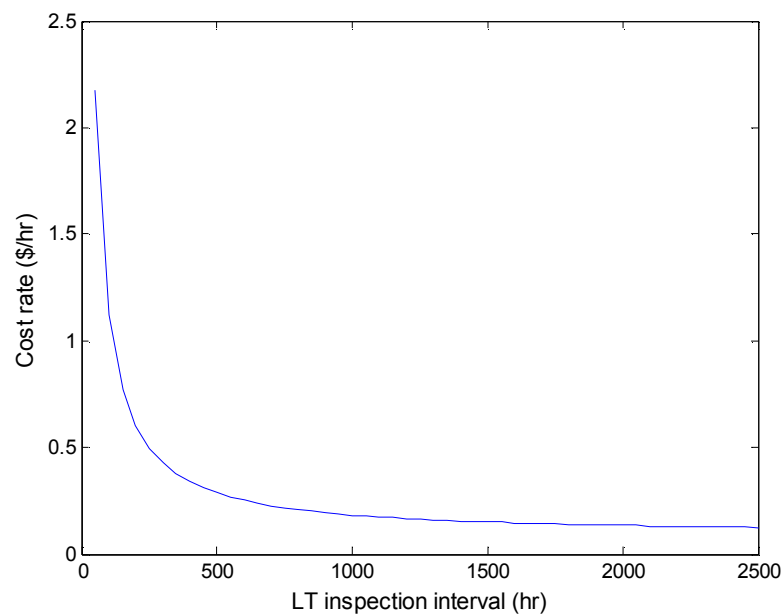

Figure 32: Cost Rate under Exponential Distribution for Cost Setting 1

The second cost setting seems exaggerated to show how the optimal policy should be changed according to the situation when the inspection cost is very cheap. The cost setting forces more frequent LT inspections, and the resulting inspection provides high availability from the proposed methodology. If operators remain the same LT inspection interval as in cost setting 1 then they have to pay approximately 0.01 dollars per unit time. The difference is very small in this example due to the property of exponential distribution, but the difference would be considerable if other IFR distribution is used.

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The cost rate is depicted in Figure 33 to show the increase in convexity of the objective function from the cost setting 2. Even though with the exaggerated cost setting, the convexity is not strong as the reasoning above.
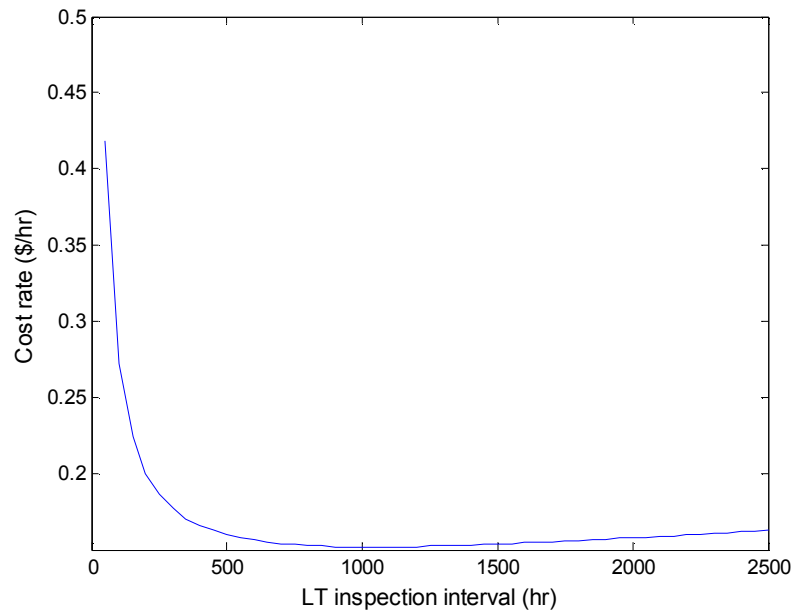
Figure 33: Cost Rate under Exponential Distribution for Cost Setting 2

The analysis verified the consistency between the results of proposed methodology and that of traditional approach under typical cost setting. If different cost settings are applied then the results from the proposed methodology is more cost efficient, since it takes account for the cost incurring during the operation. Following sections will address the same problem under strictly IFR distribution.

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6.3 *Maintenance Scheduling under Strictly IFR Distribution*

The previous analysis is examined under the constant failure distribution. There are data sources to estimate exponential distribution for the electronic component lifetime from the memoryless property [28]. Beside of the memoryless property of exponential distribution, the resultant failure follows Poisson process, and it is relatively easy to be obtained the parameter for the Poisson process by its definition [38, 83]. Nevertheless, the constant failure rate does not fully explain the behavior of most components which do have increasing or decreasing failure rate as time passes. The component tends to decrease the failure rate in the earlier phase by adjusting itself to the operational environment. This phase is called as Burn-in phase before reaching constant failure rate. The continuous workload damages the component to increase the failure rate, and this is described as Wear-out phase. By combining three phases, the more general curve is generated, and it is widely known as Bathtub Curve [105]. Typically, the manufacturers realize about the early burn-in phase, and they pre-run the component to eliminate the component having earlier failure. Therefore, most components can be modeled as having constant failure rate and increasing failure rate as operation time passes. This is, indeed, the characteristic of the IFR distribution, and the optimal maintenance problem is suitable under the failure rate having constant and increasing failure rate.

The word 'Strictly' is used in this section to rule out the constant failure case. Weibull distribution of kappa value greater than 1 can be included in this category. Like Weibull distribution, there are many two-parameter univariate lifetime distributions that can represent strictly IFR distribution. As mentioned above, it would be nice to have flexibility over the distribution if it can represent the more general distribution like

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Bathtub Curve. Most of two parameter univariate distributions cannot have a flexible increasing slope due to the degree of freedom by two parameters. There are three parameter univariate distributions which have more flexibility in properties. Generalized Pareto distribution and Increasing Decreasing Bathtub (IDB) distribution are the examples of such distributions. Pareto distribution only has DFR property, so generalized Pareto distribution expands the Pareto distribution to represent IFR distribution with the parameter setting [26]. IDB distribution advocated by Hjorth is also capable of generating the Bathtub Curve [41], and the distribution is used in this analysis to represent delayed wear-out phase in Bathtub curve, meaning that the increasing slope is initially flat and gradually increased. There are also mixture models that can represent the Bathtub curve [106], but IDB is examined in this thesis. The functional form of the IDB distribution is listed in Table 22.

Table 22: Functional Form of IDB Distribution

$f(t)$	$S(t)$	$h(t)$
$\dfrac{(1+\kappa t)\delta t + \gamma}{(1+\kappa t)^{\gamma/\kappa+1}}e^{-\delta t^2/2}$	$(1+\kappa t)^{-\gamma/\kappa}e^{-\delta t^2/2}$	$\delta t + \dfrac{\gamma}{1+\kappa t}$

The probability density function and the survival function may seem complicated from the three parameters. Simulation by inverse transformation can be applied to generate random variables of IDB distribution, since the closed-form of the survival function is available. The uniform random number from 0 to 1 is generated, and the corresponding IDB random variable, t, is numerically calculated from the survival function, because it does not have closed expression for inverse transformation. The comparison between the exponential and IDB distribution is depicted in Figure 34.

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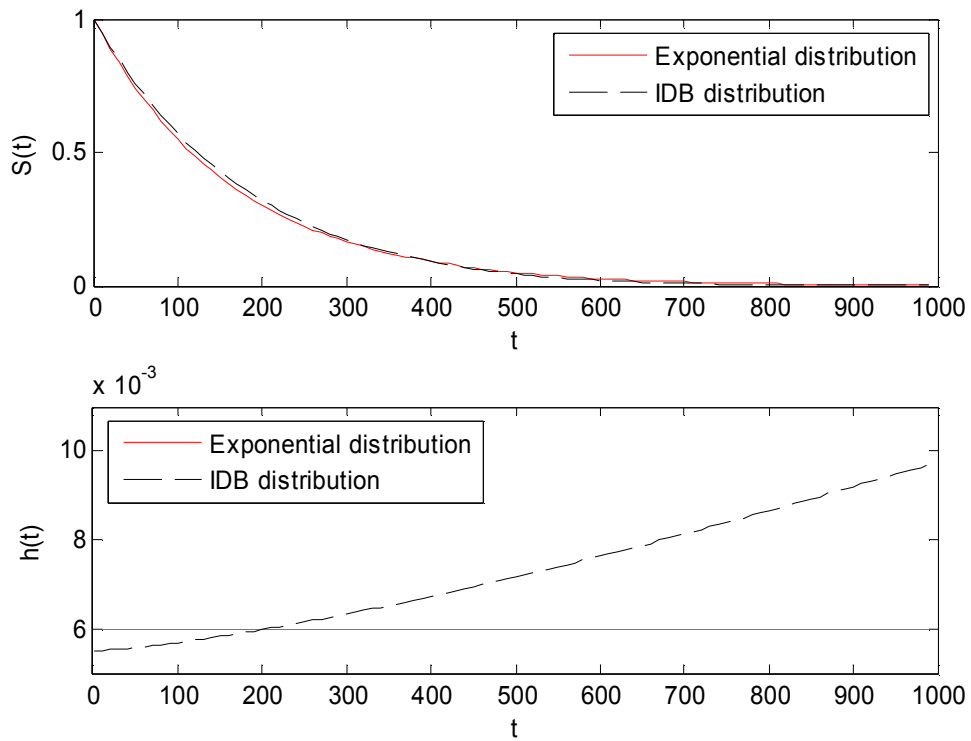
Figure 34: Comparison between Exponential and IDB Distribution

The expected life time of exponential distribution and IDB distribution are set as equal to have the same setting. The expected value of lifetime of IDB distribution can be obtained from equation (49) [41].

$$E[T] = \int_0^\infty S(t)dt = \frac{1}{\kappa} I\left(\frac{\delta}{\kappa^2}, \frac{\gamma}{\kappa}\right)$$

$$\text{where } I(a,b) = \int_0^\infty \frac{e^{-at^2/2}}{(1+t)^b} dt$$

(49)

The factor setting for IDB distribution to represent exponential distribution is $\delta = \kappa = 0$. It is observed that γ factor can be regarded as having a similar role of constant failure rate in exponential distribution. There are infinite numbers of

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combination to satisfy the expected lifetime of IDB distribution to be same as the one of exponential distribution. Setting γ factor closes to the failure rate from the exponential case and adjusting rest of factors to have IFR property enable one to find the proper factor combination to have plot as shown in Figure 34. The integral in equation (49) is numerically solved under Trapezoid scheme.

The parameter setting of $\delta \geq \kappa\gamma$ provides the failure rate of the component starts from non-zero value and strictly increases as time passes. Typically, the hazard rate of two parameters univariate strictly IFR distribution initiates from 0 which may lead lesser number of failure for a short period of time when it is compared by the exponential case. This can result in a counterintuitive observation if one simply changes exponential distribution by two parameters strictly IFR distribution, such as Weibull distribution, to expect to have more failures for a given period. On the other hand, it can be observed from IDB distribution, simple change from exponential distribution will result definite increase in expected number of failure for most of operational time. Therefore, the outcome from IDB distribution can be directly compared with the previous result by exponential distribution.

The same DOE table from the exponential case is used for this analysis, since there is only one decision variable, i.e. LT inspection interval. The examples of RSEs constructed from MC simulation are expressed in equation (50). Furthermore, the actual versus predicted plot is depicted in Figure 35.

$$
\begin{aligned}
ST &= 4.84 \times 10^{-10}\ Ti^2 + 2.70 \times 10^{-5}\ Ti + 1.004 \times 10^{-3} \\
LOTC &= 2.55 \times 10^{-9}\ Ti^2 - 4.12 \times 10^{-6}\ Ti + 1.35 \times 10^{-5}
\end{aligned}
\tag{50}
$$

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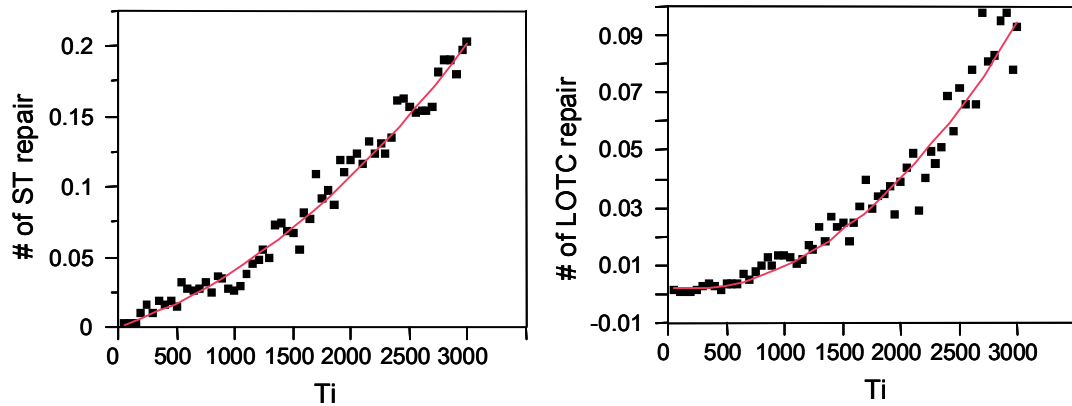
Figure 35: RSEs Fit Plot under IDB Distribution

The R-square value for reliability measures in equation (47) is above 0.95, and the prediction error is acceptably small as illustrated in Figure 35. It can be also observed that the expected number of maintenance is increased by the strictly IFR distribution. For instance, the maximum value of expected number of LOTC repair from the constant failure distribution is around 0.032 from Figure 31. This metric is increased up to 0.095 for the strictly IFR distribution case. As mentioned above, the failure rate (hazard rate function) for strictly IFR distribution start from the rate little smaller than the failure rate of the exponential distribution, but it gradually increases as operation time increases.

The same cost setting is applied as the constant failure case for the numerical optimization. The LT inspection interval from the traditional approach is not addressed in the result table. One can numerically solve for LT inspection interval from equation (22) based on the functional form of IDB distribution. It is expected that the probability density function of IDB distribution is complicated enough to require a numerical scheme for the integral calculation in equation (22).

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Table 23: Optimization Result under IDB Distribution

Maintenance cost setting	Industrial approach	Proposed
$c_{fLOTC} = 2500$ $c_i = 100$	N/A	$T_i = 2220$ Cost rate $= 0.1291$ Availability$=0.9995$
$c_{fLOTC} = 10000$ $c_i = 10$	N/A	$T_i = 673$ Cost rate $= 0.1360$ Availability$=0.99971$

The resulting LT inspection interval from the traditional approach should be same as the result from the proposed methodology for the first cost setting, since the constraint is active. The LT inspection interval under IDB distribution for the first cost setting is little smaller than the constant failure rate case, even with the active constraint. This can be explained by the strictly increasing failure rate which results in more frequent failures as operational time passes. The exaggerated cost setting also has a shorter LT inspection interval prevent from the system failure.

Figure 36 shows the behavior of the cost rate as function of LT inspection interval for cost setting 1. The plot from the exaggerated cost setting gives a good look of the convexity. The relatively low inspection cost pushes inspection interval to become shorter, and the large LOTC CM cost provides the steeper slope to represent increase in total as LT inspection interval increases.
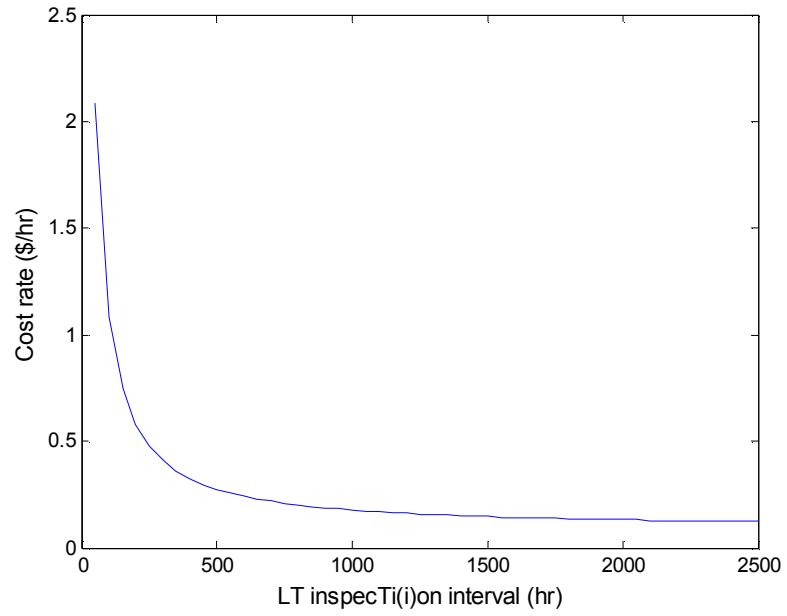
Figure 36: Cost Rate under IDB Distribution for Cost Setting 1
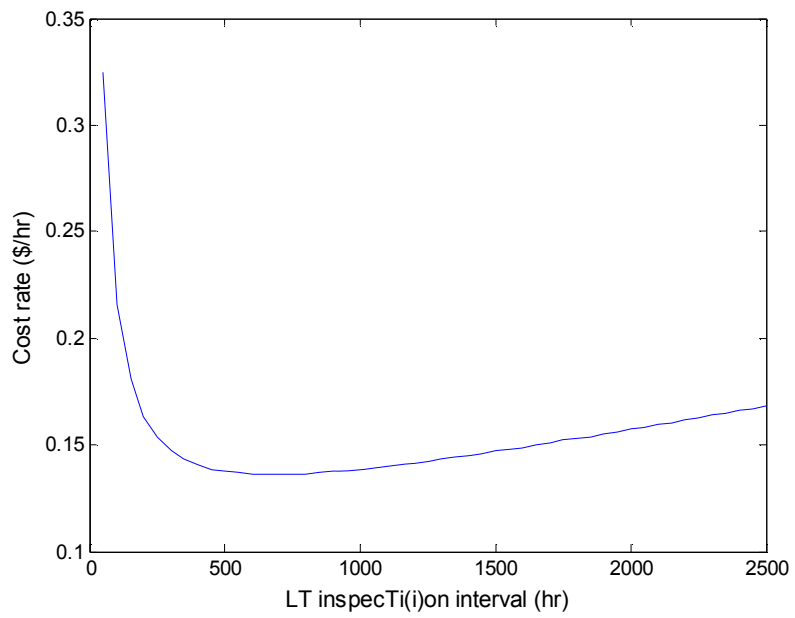
Figure 37: Cost Rate under IDB Distribution for Cost Setting 2

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6.4   *Maintenance Scheduling Considering Economic dependency*

So far TLD of FADEC system is examined under the optimal maintenance problem without considering any sources of dependencies. The traditional approach used in industries assumes exponential distribution for lifetime and repair time to set up the problem under Markov model. It is observed that different cost setting may induce sub-optimal solution from the traditional approach. The lifetime is generalized by IDB distribution to represent the reality. Because of strictly IFR property of the IDB distribution, the resulting cost rate tends to more convexity than the exponential case.

In this section, dependency is added to the strictly IFR distribution assumption. The dependencies among components become stronger as the number of components in the system grows. The dependencies, in this analysis, are considered from failure and economic standpoint. The failure dependency indicates that the failure of one component influence over the rest of working component. It can be modeled as shared-load or standby system. The mathematical property of shared-load system can be discussed under the exponential distribution which assumes that the failure of one component increase the failure rate of other working component [49, 87]. More recently, the optimization is constructed to the shared-load system to obtain the optimal workload [20]. It is realized that the failure dependencies are already included in the previous analyses. The parameters of the lifetime distributions from ARP 4761 include the increase in the failure rate in redundant components, if there is a failure in the same category [85]. The Markov model can be used for the failure dependency since each transition denotes the conditional probability from one state to another. If there is no failure dependency then the failure rate of the original component and the one for the redundant component

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should be same.

There is another type of dependency from the economic standpoint. Economic dependency can be observed from our daily life. For example, we get coupons for getting a discount on tire inspection, when engine oil is changed. If one is to change the oil and inspect tire separately then the total cost would be higher than the cost of joint operation at one time. Physical reasoning for the economic dependency is from the common operation during the maintenances. The workers should go under the car for the oil change, and it also requires lifting the car for the tire rotation. The common action for different maintenance provides cheaper cost as formulated in equation (51).

$$\begin{aligned} &C_1, C_2 < C_{1\&2} < C_1 + C_2, \ C: \text{ maintenance cost} \\ &w_1, w_2 < w_{1\&2} < w_1 + w_2, \ w: \text{ maintenance time} \end{aligned} \tag{51}$$

Equation (50) also addresses the time saving from the joint maintenance by economic dependency.

The economical dependency covered by the maintenance policy is called as an opportunistic maintenance. There are several types of opportunistic maintenance policies, but they have a common concept that while performing CM for the failed component, it is may be efficient to do the PM for component which is operating. The efficiency would be captured from equation (51) that less cost and less time is spent if the system is planning to run for a long time as most of analysis is based on long-run average measure. One type of opportunistic maintenance policies, (t_i, T) policies is introduced below [97].

If subsystem 0 fails at any time before T, perform imperfect repair

If subsystem i fails when the age of subsystem 0 is in the time interval of $[0, t_i]$ then

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replace subsystem i (perfect CM for subsystem i)

If subsystem i fails when the age of subsystem 0 is in the time interval of $[t_i, T]$ then replace subsystem i and perform perfect PM for subsystem 0

If subsystem 0 survives up to time T then do perfect PM for subsystem 0

The (t_i, T) opportunistic maintenance policy links maintenance of subsystem 0 with other subsystems to include economic dependence. The policy is very suitable for the system which has one critical component and other less important components. For example, the critical component would be the component with higher failure rate. For another instance, the component, whose failure result in system failure, can be considered as the critical system. Those assumptions can be applied to the practical system since most systems have one major component that influences on the system behavior.

If (t_i, T) opportunistic maintenance is applied to TLD of FADEC system then the t_i would be the trigger age of LT component, whenever the ST component is repaired. For example, the ST component is failed and it is planned to undergo CM for the component. (t_i, T) opportunistic maintenance allows an operator to compare the age of LT component with the predetermined trigger age of LT. If the trigger age is larger than the age of LT component then the operator assumes that the LT system is young enough not to perform any maintenance. If the trigger age is smaller than the age of LT component then there would be the opportunistic maintenance, i.e. CM for the ST component and maintenance for the LT component. The operator needs to check the state of LT component at the time of an opportunistic maintenance. If the LT component is failed then CM is performed on the component. If the LT component is operation at that

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time, PM is performed to make the system as good as new. Either case, the system restores back to the initial state to have the renewal cycle by the opportunistic maintenance. The example of opportunistic maintenance cost setting is formulated in equation (52). The PM cost for the LT component is neglected in this equation by assuming inspection itself can be regarded as PM for the working component.

$$
\begin{aligned}
c_{om-CM} &= 0.7 \times \left(c_{fST} + c_{fLT} + c_i \right), \text{ if LT is failed} \\
c_{om-PM} &= 0.7 \times \left(c_{fST} + c_i \right), \text{ if LT is working}
\end{aligned}
\tag{52}
$$

If the opportunistic maintenance has not been performed by go/no-go criterion of trigger age then the maintenance at T restores the system to the initial state. This case is same as the one decision variable of LT inspect interval. The optimal maintenance problem of TLD of FADEC system is expressed in equation (53) to include the expected number of opportunistic maintenance at different LT states.

$$
\begin{aligned}
\min : \ &\frac{\text{maintenance cost per cycle}}{\text{renewal cycle}} \\
&= \frac{c_{fST}ST + c_{fLT}LT + c_{fLOTC}LOTC + c_{omCM}omCM + c_{omPM}omPM + c_i}{D} \\
&s.t. \text{ availability constraint}
\end{aligned}
\tag{53}
$$

Two additional reliability measures, i.e. *omCM* and *omPM*, would be formulated from RSEs as function of trigger age (T_{om}) and LT inspection interval (T_i). DOE table from previous analysis with one decision variable case should be modified to include additional decision variable. The standard DOE table is not recommended to be used in this example, since the trigger age is constrained by the LT inspection interval. The customized DOE table is generated to only select the non-trivial sample points for the

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analysis. The creating of DOE table is similar to the case for the (τ, T) maintenance policy. After simulation results are gathered for reliability measures, RSEs are constructed. Examples of RSEs are shown in equation (54).

$$
\begin{aligned}
ST = & -5.781 \times 10^{-3} + 3 \times 10^{-6}\ T_i\ + 4.1 \times 10^{-5}\ T_{om} \\
& + 8.9 \times 10^{-9}\ (T_{om}\ -\ 676.82)\ (T_{om}\ -\ 676.82) \\
omPM = & -4.31 \times 10^{-4} + 2.3 \times 10^{-5} T_i + 8.6 \times 10^{-9} T_i^2 - 2.5 \times 10^{-5} T_{om} \\
& -1.4 \times 10^{-9} (T_{om} - 676.82)(T_{om} - 676.82) - 6.9 \times 10^{-9} T_i (T_{om} - 676.82)
\end{aligned}
\tag{54}
$$

The expected number of ST repair mostly depends on the trigger age, and the expected number of an opportunistic maintenance when the LT component is operation depends on both decision variables. Both RSEs result in R-square values above 0.98 to have very good fit, and the response for each unknown function is illustrated in Figure 38 and Figure 39, respectively.
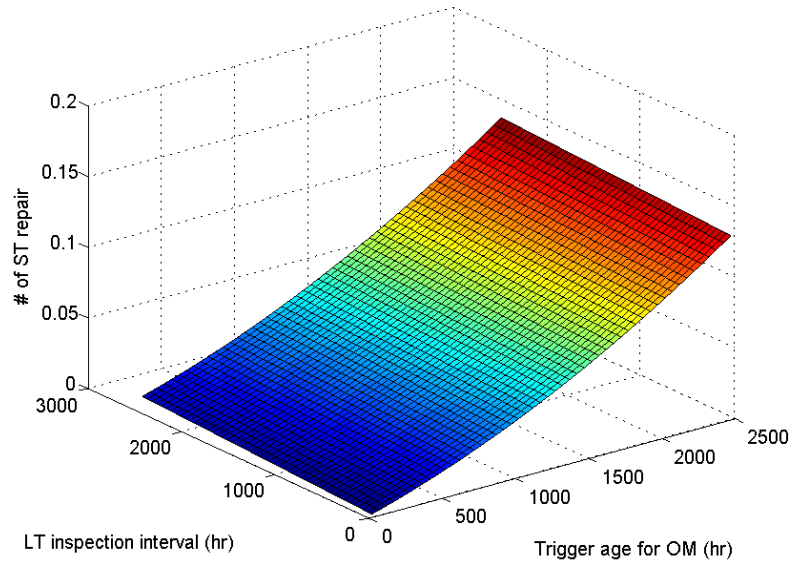
Figure 38: RSE of ST with respect to Trigger Age and LT Inspection Interval

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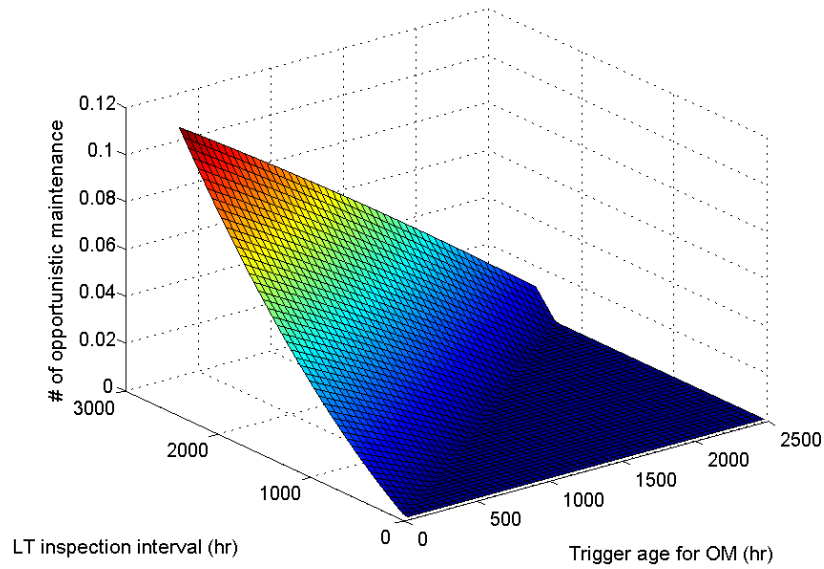
Figure 39: RSE of omPM with respect to Trigger Age and LT Inspection Interval

The expected number of an opportunistic maintenance when LT is operation is consistent with the common sense that shorter trigger age provides more frequent maintenance. If the difference between the trigger age and LT inspection becomes smaller, there would be lesser chance of performing the opportunistic maintenance based on the given maintenance policy.

The optimal maintenance polices for different cost settings are listed in the following table.

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Table 24: Optimization Result under Economic Dependency for 70% OM Cost

Maintenance cost setting		Proposed
$c_{fLOTC} = 1500, c_i = 200$	70% OM cost	$T_{om} = 696, T_i = 696$ Cost rate = 0.3856 Availability=0.9995
$c_{fLOTC} = 2000, c_i = 150$	70% OM cost	$T_{om} = 274, T_i = 696$ Cost rate = 0.3190 Availability=0.9995
$c_{fLOTC} = 10000, c_i = 10$	70% OM cost	$T_{om} = 0, T_i = 330$ Cost rate = 0.1458 Availability=0.99957

The 70-percent cost efficiency is assumed for opportunistic maintenance as in equation (51). The RSE for availability turns out to be the function of LT inspection interval alone, and the LT inspection interval is to set to satisfy the given availability constraint. That is the reason why the inspection interval is constant when the constraint is active. The result from cost setting 2 suggested that increase in system CM cost forces the frequent opportunistic maintenance, since trigger age of LT becomes smaller. The optimal policy of exaggerated cost setting 3 is opposite from the cost setting 1 that it is always optimal to perform an opportunistic maintenance, whenever CM for ST component is performed. Beside the results in the table above, different settings for opportunistic maintenance cost are analyzed. As the percentage of opportunistic cost increase, the trigger age tends to increase as in Table 25.

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Table 25: Optimization Result under Economic Dependency for Varying OM Cost

Maintenance cost setting		Proposed
	20% OM cost	$T_{om} = 0, T_i = 696$ Cost rate = 0.3102 Availability=0.9995
$c_{fLOTC} = 2000, c_i = 150$	70% OM cost	$T_{om} = 274, T_i = 696$ Cost rate = 0.3190 Availability=0.9995
	150% OM cost	$T_{om} = 696, T_i = 696$ Cost rate = 0.3210 Availability=0.9995
	20% OM cost	$T_{om} = 0, T_i = 368$ Cost rate = 0.1145 Availability=0.99957
$c_{fLOTC} = 5000, c_i = 10$	70% OM cost	$T_{om} = 0, T_i = 352$ Cost rate = 0.1202 Availability=0.99957
	150% OM cost	$T_{om} = 18, T_i = 327$ Cost rate = 0.1291 Availability=0.99957

The observation from the exaggerated cost setting is very interesting. The availability constraint is not governing the optimal solution due to relative small inspection cost. As percentage of OM cost increases, LT inspection interval tends to decrease. It is obvious that the decrease in the inspection interval makes the chance of OM decrease to incorporate the higher OM cost as the percentage grows. The objective function is depicted in following figures for the result in Table 24.

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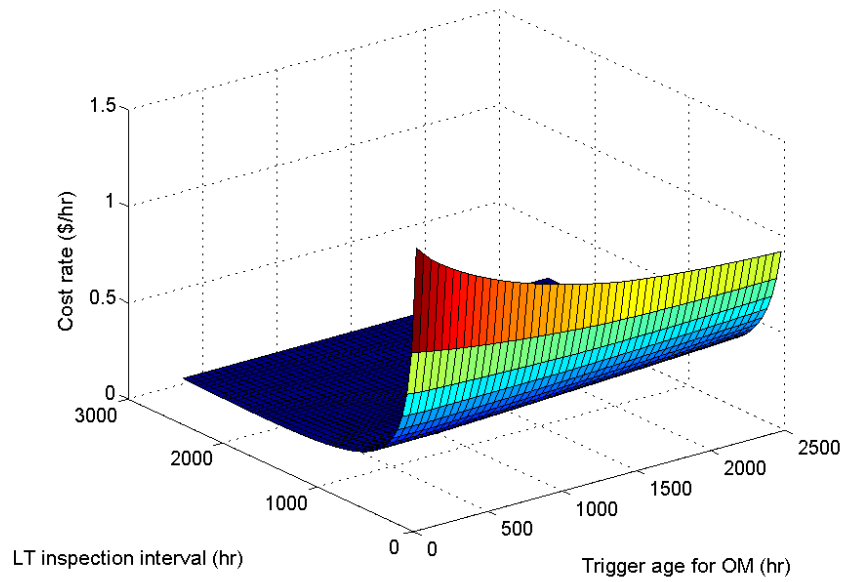
Figure 40: Cost Rate under Economic Dependency for Cost Setting 1
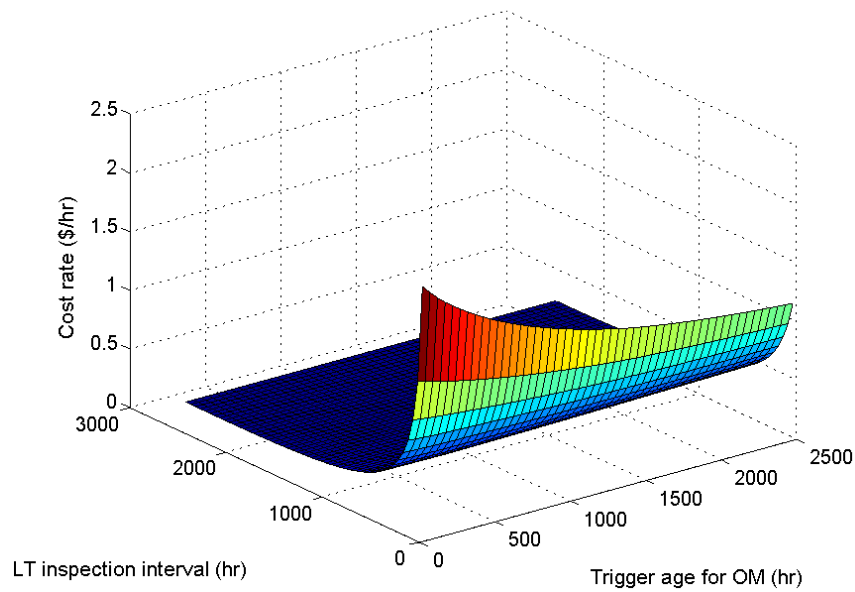
Figure 41: Cost Rate under Economic Dependency for Cost Setting 2

Two plots look similar but the second one show the convexity with respect to trigger age.

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The convexity of the objective function is well illustrated in Figure 42 for the cost setting 3.
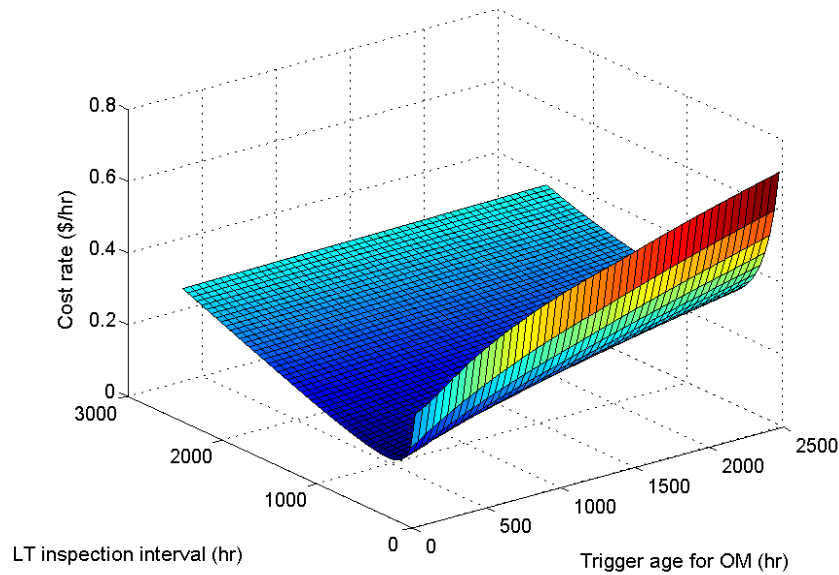
Figure 42: Cost Rate under Economic Dependency for Cost Setting 3

It is also found that different cost settings result in different combination of trigger age and LT inspection interval. Typically, there is a strong relationship between the maintenance cost settings and the optimal maintenance policies, if there is only one decision variable. It is concluded that the strong relationship diminishes for the maintenance problem under economic dependencies, and this indicates the complexity of the optimal maintenance problem when dependencies are included.

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6.5   Maintenance Scheduling for Multi-state Model

Previous analyses are performed under the single-state model. The single-state model is illustrated in Figure 29. The notation of 'single-state' is defined as the model which has only one state to reach the system failure state, i.e. LOTC state. It is observed from the diagram that there is either ST state or LT state before reaching LOTC state from FU state. If there are multi components in either ST state or LT state, the previous analysis can be modified slightly to simulate each component in the same category. The steady state LOTC rate for the multi component single-state model is expressed below

$$\lambda_{LOTC} = \frac{\lambda_{ND} + \sum_i \dfrac{\lambda_{ST_i}\left(\lambda_{STT_i} + \lambda_{ND}\right)}{\mu_{ST} + \lambda_{STT_i} + \lambda_{ND}} + \sum_i \dfrac{\lambda_{LT_i}\left(\lambda_{LTT_i} + \lambda_{ND}\right)}{\mu_{LT} + \lambda_{LTT_i} + \lambda_{ND}}}{1 + \sum_i \dfrac{\lambda_{ST_i}}{\mu_{ST} + \lambda_{STT_i} + \lambda_{ND}} + \sum_i \dfrac{\lambda_{LT_i}}{\mu_{LT} + \lambda_{LTT_i} + \lambda_{ND}}} \tag{55}$$

Therefore it is required to group the component into ST or LT for the dispatch interval to generate the common repair policies for components in same category by MC simulation. Only few lines of code should be added in MC simulation to include more components in each category.

The problem, however, becomes complicated if one is going to have the model for multi-state from industrial approaches. The multi-state model assumes that there may be more than one state to reach the final state, i.e. LOTC state. For instance, there are 3 components (A, B and C) under ST dispatch. Each component has one redundant component for the safety. The single-state assumes that if component A fails then the next state would be system failure or system working. The system is failed by the

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redundant component A is failed before the repair of the original component. Under most cases, system becomes working because the expected repair time of component A is relatively smaller than the expected lifetime of the redundant component. This transition can be different in multi-state model that the next state can be among the 4 cases; failure of B, failure of C, failure of the redundant component A to have system failure and system repair. If industrial approaches under CTMC are modeled for the problem than the process can easily suffered from the curve of dimensionality. Total n components may result in 2^n combination before reaching the LOTC state. Industrial approaches may handle this problem in two ways. One way is to set the transition to stop at dual-state or triple-state to limit the dimensionality of Markov model. The approach assumes that there will be repairs in failed components before additional failure occurs. Even with the dual-state for multi component case, constructing steady state LOTC rate as in equation (54) involves solving non-trivial system of equations. Another way to handle the multi-state model is to set the go/no-go criterion for the model. It is found by the operator that if repair time of LT is 1000 hours or less then a sing state Markov model can result less than 1% error when compared with dual-state case [32]. For out example, LT dispatch interval of 1250 hour is used, so single-state model has an accurate result.

The curve of dimensionality is not the problem for the proposed methodology. It is discussed in Chapter 4 that MC simulation in the proposed methodology is based on the algorithm from Kim, at el which simply take minimum of simulated lifetime to check for the transition from one to another. The simple algorithm allows including full state to cover all possible transitions occurred during the operation. The computational time is same for whether it is single-state or multi-state model, because the computational time is

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mainly determined by the number of runs, maximum run time and number of sample points in DOE table. These input parameters are set as equal for any-state model, and the MC simulation used in the proposed methodology in previous examples, indeed, is based on the full state model to capture any chances of crossover failures occur before the repair. The small error from the result for cost setting 1 in Table 21 can be reconsidered as the error between single-state and multi-state model. The error is small due to the small LT dispatch interval (1250 hours) as the operators suggest.

There is another consideration if multi-state model is to be used. According to the current paper [77] for TLD of FADEC system, the category of components may change based on the failed states. For example, if components in LT states are failed before the repair is performed. There may be the combination of such LT failure makes system vulnerable enough to perform maintenance immediately. In this case, LT components become ND components based on the combination of failures. Example of such combination and the resulting action is discussed in [78]. The traditional approach cannot handle such situations even under the multi-state model using CTMC. Introducing new states may give an insight for the problem, but the dimension is already large for the multi-state model. On the other hand, Prescott (2005) addresses safety modeling for TLD of FACEC system by MC simulation and concluded that MC simulation is flexible with such cases that simply imposing rules for different maintenance action can solve the problem [78]. The similar scheme can be utilized in MC simulation of the proposed methodology to capture general cases.

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6.6 Summary of Maintenance Scheduling for TLD of FADEC System

TLD of FADEC system is review under the optimal maintenance problem. It is mentioned in section 5.2 that simple cost distribution can modify the problem to capture the inspection cost and other operational cost. In this chapter, more general setting is modeled to achieve the optimal maintenance.

It is required to have the same result as the industrial approach under the same assumption. The constraint of steady state LOTC rate given regulars determines the LT dispatch interval. If components in LT category has indicator as the components in ST category, the operator can use this dispatch interval directly to perform CM. Unfortunately, it is addressed that there is no such device for LT components since the failure rate of the component is relative low than that of ST component. Periodic inspection is performed regularly to check the LT component, and there is equation to relate LT inspection interval with time since failure of LT (LT dispatch interval) in terms of expectation. It is the traditional approach to iteratively solve for the inspection interval for given time since failure of the component. It is also mentioned that operators impose subjective decision to use time since failure of LT from regulation of LOTC failure rate or not. This is the observation from the practical application that the inspection interval from LOTC constraint sometimes induce more cost depending on the maintenance cost setting which tends to be fluctuated based on the unforeseen events. That is why, operator make some judgments to calibrate the input to obtain the inspection interval. If the subjective decision making turns out to be reasonable then the maintenance policy is optimal. Otherwise, it may induce undesirable increase in maintenance cost or result in very unreliable system not to meet the regulation.

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The proposed methodology tries to solve the limitations from traditional approach by obtaining the maintenance policy under optimization problem. If the optimization is correctly modeled than the resulting policy would have minimum cost while achieving the requirement. Therefore, LT inspection for TLD of FADEC system is solved by setting long-run average cost rate as the objective function and long-run average availability as constraint for the optimization problem. The long-run average measure is used to capture the infinite horizon of the operation, not to mislead by the predetermined finite horizon.

It is observed from the previous optimization setting that the availability constraint is not stated from the regulators. As mentioned regulation is only stated for the stead state LOTC failure rate since it is easy to have the closed form formula from given rates, and it is not depends on the feedback rate which tends to be very arbitrary to make the closed loop Markov model [32]. Fortunately, it turns out that there is one to one correspondence between steady state LOTC rate and availability from simple mathematical transformation of Markov model. After corresponding availability is calculated the optimization is solve to yield the maintenance policy which is close to the one from the traditional approach. The availability constraint is active for the proposed methodology to yield the same result as the traditional case. The cost setting, which assumes relatively small inspection cost and high cost for system repair is considered for another scenario. The resulting LT inspection for this case by proposed methodology is consistent with the intuition that lesser inspection cost may lead the inspection interval more frequently.

The benchmark modeling under exponential distribution is performed to validate

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the proposed methodology. It is observed that constant failure rate is not sufficient to represent the lifetime of the real application. The 3-parameter distribution of IDB function is utilized to capture the strictly IFR distribution. Such strictly IFR distribution is expected to have more frequent failure than exponential case to force LT inspection interval be shorter to minimize the cost rate. The same procedure of MC simulation and RSE construction is performed under IDB distribution. Only difference occurs for generating the random variable for IDB distribution, since inverse transformation of the survival function of IDB distribution is not possible. Numerical root finding based on Bisection method is used to generate random numbers for IDB distribution [79]. The response of the objective function with respect to LT inspection interval shows more convexity from the strictly IFR distribution. If the operator were to be conservative with the maintenance schedule from constant failure case then there would be big difference in cost rate under cost setting 2. It should be noted that the objective is the cost rate so the operational duration should be multiplied to cover the total cost. Moreover, the cost setting itself is arbitrary for this analysis, it can be anticipated that the deviation from the optimal solution under strictly IDB distribution becomes higher if operators assume constant failure distribution.

Next, the dependency is added to the previous model, since it is observed from the statistical data that more components in the system would increase the dependencies from failure mode and economic standpoint. Failure dependency is modeled under the increase in failure rate of state transition of redundant component given present of failure in original component. The parameters from ARP-5107 assume the failure dependency [32], but it can be easily implemented in MC simulation module for the proposed methodology

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to change the failure rate depending on the other component states. Another type of dependency is from economic aspect that common operation in different maintenance may require lesser time and cost than the case of separate maintenance. To capture the economic dependency, the new decision variable of trigger age for LT opportunistic maintenance is introduced. The trigger age is compared with the age of LT component, whenever CM for ST failure is performed. If the trigger age is larger than age of LT component then it is assumed that the LT component is young and it is optimal to skip the opportunistic maintenance. If trigger age is smaller than the age of LT component, there would be the opportunistic maintenance based on the current state of LT component. It is found out that the optimal maintenance policies differ based on the opportunistic cost setting and inspection cost. Furthermore, it is observed from the analysis that the problem considering economic dependency increases the complicity in relationship between decision variables and the figure of merits not to conclude any simple rules for maintenance scheduling.

We have examined the TLD of FADAEC system from very basic assumption of constant failure to advance area which deals with economic dependency to have more accurate model to represent the reality. It is also discussed that the model can be expanded to include multi units in each category and multi-states between FU state and LOTC state. The tradition approach has limitations in incorporating the multi unit / multi-state modeling by dimensionality, but it is addressed that the MC simulation in the proposed methodology is flexible enough to incorporate such improvements.

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CHAPTER 7   CONCLUSION

Establishing an optimal maintenance policy for a modern aerospace system is important with regards to its operation cost and overall safety of the system. The outcomes of the present research demonstrate the effectiveness of applying relevant numerical techniques to solve the optimal maintenance problem to satisfy a desired level of system availability while inducing the minimum maintenance cost per operation hour. The TLD of FADEC system is studied as a benchmarking example to showcase how optimal maintenance policies can be obtained for a complex, real-world engineering system without resorting to oversimplifying its mathematical or structural aspects.

7.1   *Qualitative Benchmarking of the Proposed Methodology*

What follows is a better summary of the contribution of this work by a qualitative benchmarking of the proposed methodology for maintenance scheduling. Five maintenance approaches are selected for the purposes of comparison spanning the domains in between industry and academia as listed in Table 26. Trial & Error is the simplest approach to search for optimal maintenance policies. The arbitrary assignment of maintenance policies based on past empirical and historical data fall under this category. An example of the industrial approach would be the current inspection policy used in the TLD of FADEC system, where mathematical model, such as CTMC, is used to represent the system. RCM includes the gathering of data for the failure distributions and performing combinations of both qualitative and quantitative modeling techniques, such as Functional Failure Analysis (FFA), Failure Mode Effect and Criticality Analysis

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(FMECA) and maintenance interval determination by numerical optimization [86]. Lastly, traditional mathematical approaches to the optimal maintenance problem are compared to the methodology proposed by this thesis based on the criteria listed in Table 26.

Table 26: Qualitative Benchmarking of the Proposed Methodology

	Trial & Error	Industrial	RCM	Proposed	Mathematical
Accuracy	Bad	Acceptable	Good	Excellent	Excellent
Quantitative	Bad	Acceptable	Good	Excellent	Excellent
Rapidity	Excellent	Excellent	Acceptable	Good	Acceptable
Simplicity	Excellent	Excellent	Excellent	Good	Bad
Flexibility	Bad	Bad	Good	Excellent	Acceptable
Overall	Bad	Acceptable	Good	Excellent	Acceptable

Legend: Excellent, Good, Acceptable, Bad

A total of 5 attributes are decided to be used as the criteria, against which each approach were to be judged. Accuracy checks whether an approach can deliver an optimal maintenance policy that is the same as the analytical solution, if it exists. Quantitative is an attribute intended to measure the level of unbiased decision making, since subjective judgment can yield a maintenance policy which is not optimal. Rapidity allows the evaluation of the time needed and devoted to the setup, computation and validation of a given approach. Simplicity is also listed to measure how easy or difficult

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the methodology is to potential users. Lastly, flexibility allows comparison of one approach's broad applicability to the different types of problems to another's.

The Trial & Error approach can be easily and rapidly performed, since it does not require much a priori knowledge of the system to be studied. Previous experience in maintaining the same system is the only pre-requisite, but it is observed that the previous experience tends to lead the process not only inaccurate, but also not generalized. Therefore, this maintenance approach scores the worst overall rating.

Industrial approaches are an improvement over the Trial & Error approach with respect to their quantitative set up of the problem. It is, however, well-known that such approaches can neither sufficiently capture the different settings of maintenance cost nor other operational factors. If, for example, an operator were to consider a cost efficient maintenance policy then subjective decisions must become a part of evaluation process. Needless to say, such subjective decisions are not robust inputs, since they tend to bias the maintenance model toward either infeasible or non-optimal maintenance policies.

Currently existing mathematical approaches rely on probability theories and mathematical proofs to obtain the optimal maintenance policies. They often have closed-form solutions for simple known problems, such as single component maintenance under perfect repair or k-out-of-n system under imperfect maintenance assuming quasi-renewal processes. Nevertheless, the ensuring mathematical derivations are not trivial to make this class of approaches not practical for real-world applications that tend to have highly complex system structures, often with dependencies. Furthermore, different types of simplifying assumptions must be made during the derivation process, and it implies that such approaches will not always be universally applicable in practice.

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RCM and the proposed methodology are hybrids of the purely practical and strictly theoretical approaches. RCM begins as a qualitative approach that allows the accumulation of mathematical models, as the problem becomes more defined. In contrast, the proposed methodology first formulates a mathematical problem, and then solves it with statistical techniques. Therefore, both approaches can be viewed as attempts to narrow the gap between practice and theory.

RCM ranks well in most criteria, with the exception of rapidity. The data mining process, as well as the qualitative FFA and FMECA, are inherently time-consuming. Some of the subjective judgments implied in these qualitative analyses can have an adverse influence over the mathematical modeling that follows, thus resulting in a sub-optimal solution.

Lastly, the proposed methodology ranks the highest in both accuracy and quantitative criterion. Accuracy is guaranteed by the preliminary analysis, and the statistical approaches contained in the RSE module, along with numerical optimization, represent the quantitative aspect of decision making.

It does not, however, always have good evaluation for the rapidity and simplicity. The processing time required for the MC simulation may be considerable, if a closed form equation for inverse transformation does not exist for generating an input probability distribution, as in the IDB distribution case. For example, the application of an iterative and numerical root finding technique for the random number generation may not be practical as either the desired number of samples from the DOE table or the number of simulation cases grows.

The methodology also works low in terms of simplicity, because some basic

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knowledge of statistics is required to use the RSE module appropriately. In addition, the statistical tests, which involve the construction of as parsimonious model as possible, also demands a strong background in regression analysis from the operator.

The overall evaluation for the proposed methodology is justified on the grounds that it is rooted on a quantitatively sound foundation and allows the reaching of a solution without having to make any assumptions on distribution and oversimplification of the structures of the problem. Moreover, the only sources of inaccuracy are from the MC simulation and the regression processes, but results from preliminary analyses suggest that the inaccuracy is insignificant.

7.2   Concluding Remarks

There can be no denying that if one knows of a mathematical model that is a good fit to the real-world problem at hand, it would yield the most accurate results. The usefulness of the proposed methodology would be truly appreciated if an operator wishes to find optimal maintenance policies for a system that is arbitrary enough not to have any analytical closed-form formulas to represent it. The preliminary analyses and the TLD of FADEC example show that how MC simulation can be made flexible enough to be useful in capturing the new ideas. Moreover, the regression analysis in terms of RSEs can construct parsimonious surrogate models for the unknown functions when the system under question is complicated by dependencies.

Therefore, the proposed methodology by this thesis is suitable for constructing the maintenance schedules for the multi-unit systems whose operators are expected to increase their knowledge of the system. For example, it can be assumed that the operators will gather more data on the system with the passage of time. Such an increase in

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knowledge would tend to make the system become more detailed and complex. There may even be a case in which the initial assumption of constant failure rate for the lifetime must be changed based on newly observed data forcing the usage of new lifetime distribution. Hence, when a the fast-changing operational environment, such as maintenance cost settings, are expected or the modification of the system structure from more detail knowledge of system are to be under scrutiny, it is concluded that the proposed methodology would be a suitable process to be considered for application.

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